Announcements

→Move Thurs hours to Fri 3-4pm

- Quiz #1 Back

	Sec 011	Sec 013
AVG	4.66	4.57
Median	5	4.5

- Out of town next week; sub lecturer Wed and Fri
- → Quiz #2 // Assistance
- Today: 1.7

- Section 1.7: Exp growth and decay

- Key property: per each unit increase in the input, output gets multiplied by a constant.

- Increasing exp fn: How much do I need to increase the input to get the output to double? This amount is called the "doubling time"

- Decreasing exp fn: " " " " " " " " " " " " to halve? This amount is called the half-life.

Example A ~~see~~ 1m seedling is planted and grows at the rate of 3% each month.

②

How long does it take for the plant to grow

(a) from 1m to 2m?

(b) from 2m to 4m?

(c) from g meters to $2g$?

Soln: $H(t) = H_0 (1+r)^t$

$$= 1(1.03)^t$$

(a) At $t=0$, plant has height $H(0)=1$.

$$2 = (1.03)^t$$

$$\ln(2) = t \ln(1.03)$$

$$t = \frac{\ln(2)}{\ln(1.03)} = 23.45 \text{ months}$$

(b) $4 = (1.03)^t$

$$\ln(4) = t \ln(1.03)$$

$$t = \frac{\ln(4)}{\ln(1.03)} = 46.9 \text{ months}$$

Doubling time: $46.9 - 23.45 \approx 23.45$ months.

(c) When does plant have height g? //

Ex: After 1 minute of exposure to penicillin, 90% of a culture of bacteria has died. Assuming the pop. of bacteria decays exponentially, what is the half-life?

Soln

$$P(t) = P_0 (1+r)^t$$

Amount of bacteria at time t (in minutes)

$$= P_0 (1-.9)^t$$

$$= P_0 (0.1)^t$$

$$\frac{1}{2} \cancel{P_0} = \cancel{P_0} (0.1)^t$$

(Solve for t; when there is $\frac{1}{2} P_0$ bacteria remaining)

$$\frac{1}{2} = (0.1)^t$$

$$\ln\left(\tfrac{1}{2}\right) = t \ln(0.1)$$

$$t = \frac{\ln(\frac{1}{2})}{\ln(0.1)} \simeq 0.301 \text{ minutes} \approx \boxed{18.06 \text{ seconds}}$$

Discrete growth rates vs Continuous growth rates

- Annual Interest rate of 100%.
- After 1 year, \$1 becomes

$$P_0(1+r) = 1(1+1) = \$2$$

- Interest rate is 50%, per 6 months

$$P_0(1+r)(1+r)$$

$$= 1(1+\tfrac{1}{2})(1+\tfrac{1}{2}) = 1(\tfrac{3}{2})(\tfrac{3}{2}) = (\tfrac{3}{2})^2 = \$2.25$$

- Interest rate of $\frac{100\%}{12}$, compounded monthly

$$= 1\left(1+\frac{1}{12}\right)^{12} \approx \$2.61$$

- Interest rate of $\frac{100\%}{365}$, compounded daily?

$$= 1\left(1+\frac{1}{365}\right)^{365} \approx \$2.715$$

- Every hour? $2.71813
- Every second? $2.718277

$$e = 2.7182817\ldots$$

- When an exp function has continuous growth rate k, the formula

$$P(t) = P_0 e^{kt}$$

· Recall Discrete growth rate r:

$$P(t) = P_0(1+r)^t$$

· What continuous interest rate is equivalent to an annual discrete rate of 100%

Soln:

Discrete rate:	Continuous rate
$P(t) = P_0(2)^t$	$P(t) = P_0 e^{kt}$

Set them equal:

$$\cancel{P_0} 2^t = \cancel{P_0} e^{kt}$$

$$2^t = e^{kt}$$

$$\ln(2^t) = \ln(e^{kt})$$

$$\cancel{t}\ln(2) = k\cancel{t}$$

$$k = \ln(2) = 0.693 \ldots$$

$\Rightarrow$ A continuous interest rate of $\boxed{69.3\%}$ is equivalent to a 100% interest rate compounded once per year.