Announcements

- Final Exam

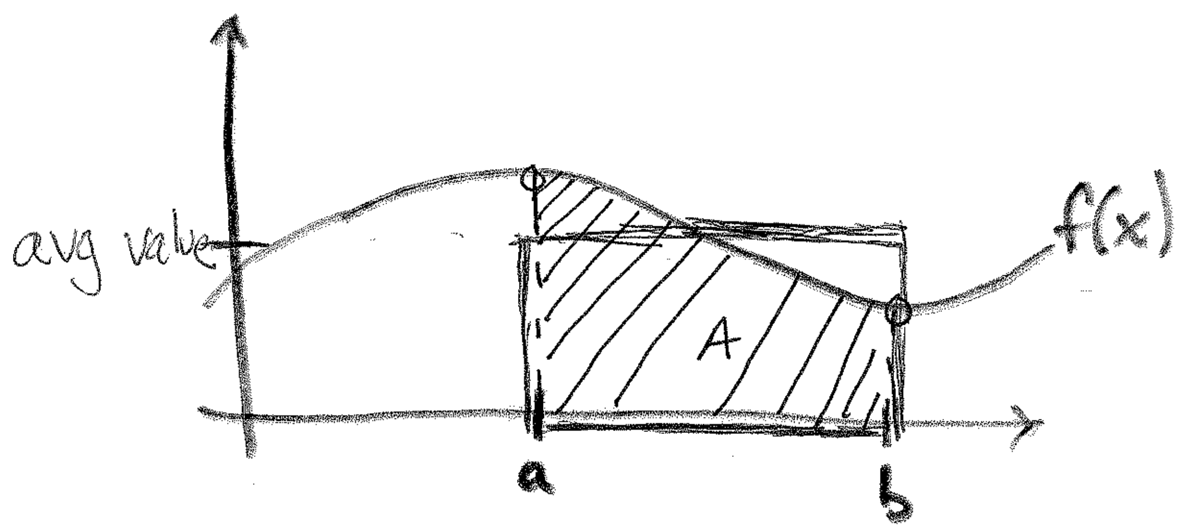
  __011:__ Fri Dec 10 9am-noon

__Ex__ 6.1

Graphical Interpretation of avg val:

$$(\text{avg val}) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$(b-a)(\text{avg val}) = \int_a^b f(x)\,dx$$



Ex 6.1 #11

- The value V of a lamp is \$225 in 1975.
- Increases 15% per year.
- $V = 225(1+r)^t$

$$= 225(1.15)^t$$

- Find the avg val of the lamp from 1975 to 2010.

Soln: $(\text{avg val}) = \dfrac{1}{b-a}\displaystyle\int_a^b V(t)\,dt$

$$= \frac{1}{35-0}\int_0^{35} 225(1.15)^t\,dt$$

$$= \frac{225}{35}\int_0^{35} (1.15)^t\,dt$$

• Find k: $1.15^t = e^{kt}$

$$\ln(1.15^t) = \ln(e^{kt})$$

$$t \ln(1.15) = k\cancel{t}$$

$$k = \ln(1.15) \approx 0.14$$

• Continuing: $\frac{225}{35}\int_0^{35} (1.15)^t \, dt$

$$= \frac{225}{35}\int_0^{35} e^{0.14t} \, dt$$

$$= \frac{225}{35}\left(\frac{1}{0.14} e^{0.14t}\right)\Big|_0^{35}$$

$$= 6.43(7.14e^{4.9} - 7.14e^0)$$

$$= 6.43(958.83 - 7.14)$$

$$= 6.43 \cdot 951.69 \approx \boxed{\$6119.36}$$

- $60e^{-0.5t}$ = Amt of crates after t months

- How many crates ~~at~~ initially?

  $\#\text{crates} = 60e^{0} = \boxed{60}$

- How many crates after 6 months?

  $\#\text{crates} = 60e^{-\frac{1}{2}\cdot 6} = 60e^{-3} \approx 2.987$

  $= \boxed{3}$

- $$\text{Avg val} = \frac{1}{6-0}\int_0^6 60e^{-0.5t}\,dt$$

  $$= \frac{60}{6}\int_0^6 e^{-0.5t}\,dt$$

  $$= 10\left(\frac{1}{-0.5}e^{-0.5t}\right)\Big|_0^6$$

  $$= 10\left((-2e^{-3}) - (-2e^{0})\right)$$

  $$= 10(-2e^{-3} + 2)$$

$$= 20(1 - e^{-3})$$

$$\approx 19.004 \approx \boxed{19}$$

Ex 1.7 #23

- The quantity of a substance decreases by 4% in 10 hours. Find its half-life.

Soln:

$$P = P_0 e^{kt}$$

$$0.96 P_0 = P_0 e^{k \cdot 10}$$

$$0.96 = e^{10k}$$

$$\ln(0.96) = 10k$$

$$k = \frac{\ln(0.96)}{10} \approx -0.00408$$

- $P = P_0 e^{-0.00408t}$

- $\frac{1}{2}\cancel{P_0} = \cancel{P_0} e^{-0.00408t}$

- $\ln\left(\frac{1}{2}\right) = -0.00408t$

- $t = \dfrac{\ln\left(\frac{1}{2}\right)}{-0.00408} \approx \boxed{169.89 \text{ hours}}$

<u>Ex</u>: Suppose a bank has a 10% interest rate compounded continuously. Bank now wants to compound annually what should the annual rate be?

<u>Soln</u>

$$P_0(1+r)^t = P_0 e^{0.1t}$$

$$(1+r)^t = e^{0.1t}$$

$$t \ln(1+r) = 0.1t$$

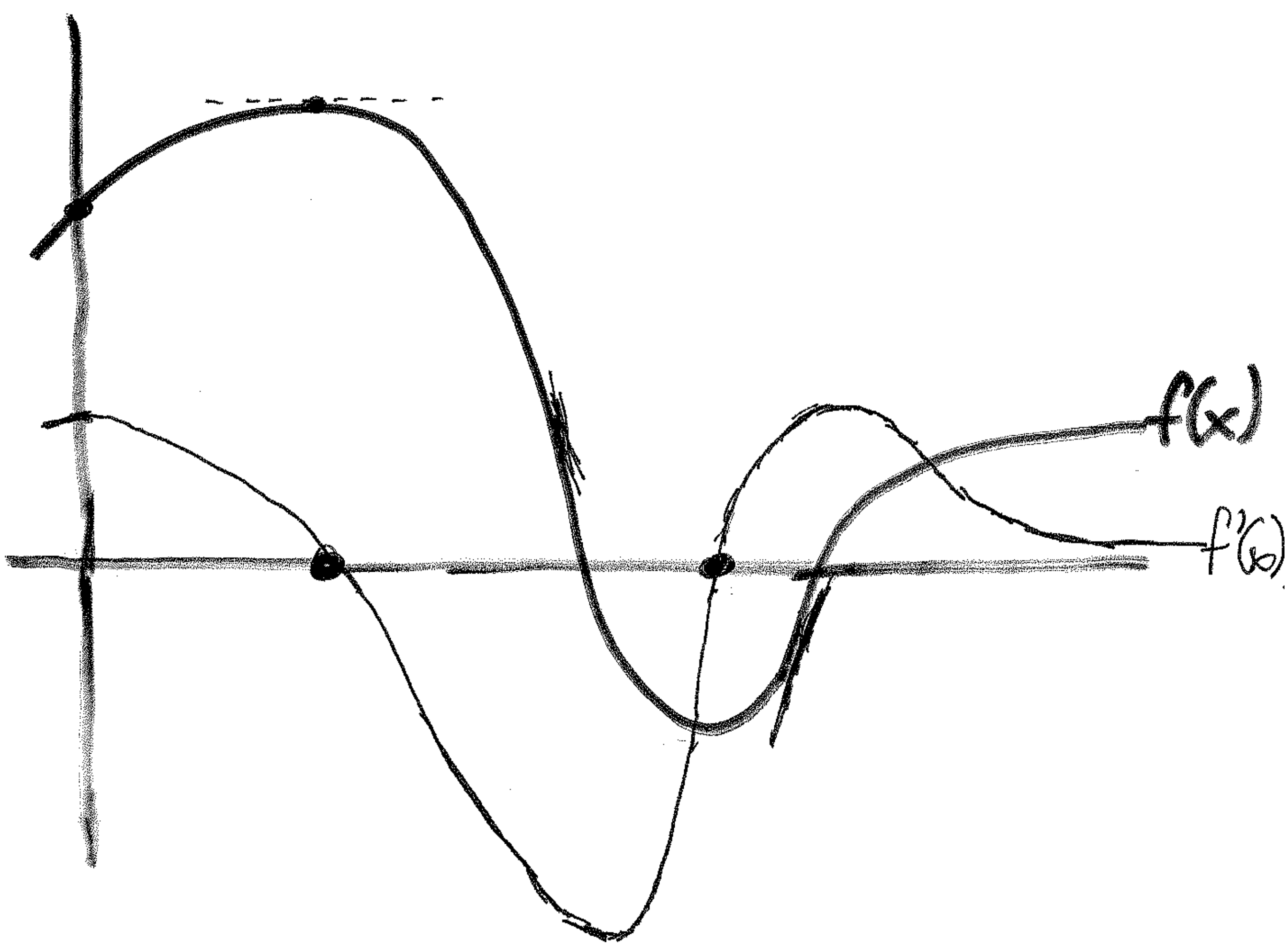
$$\ln(1+r) = 0.1$$

$$1+r = e^{0.1}$$

$$r = e^{0.1} - 1 = 0.10517$$

So the corresponding rate for annually compounding interest is $\boxed{10.52\%}$.

Ex Given the graph for $f(x)$, sketch the derivative $f'(x)$.



Announcements

<u>0013</u>: Final Exam Sat Dec 11, 2pm–5pm

<u>Ex</u>: Find the first derivative and second derivative of $f(x) = x\ln x + e^x$.

<u>Soln</u>: $f'(x) = \frac{d}{dx}\left[x\ln x + e^x\right]$

$$= \frac{d}{dx}\left[x\ln x\right] + \frac{d}{dx}\left[e^x\right]$$

$$= \frac{d}{dx}[x]\cdot\ln x + x\cdot\frac{d}{dx}\left[\ln x\right] + e^x$$

$$= 1\cdot\ln x + x\cdot\frac{1}{x} + e^x$$

$$= \boxed{\ln x + 1 + e^x}$$

$f''(x) = \frac{d}{dx}\left[\ln x + 1 + e^x\right]$

$$= \frac{d}{dx}\left[\ln x\right] + \frac{d}{dx}\left[1\right] + \frac{d}{dx}\left[e^x\right]$$

$$= \frac{1}{x} + 0 + e^x$$

$$= \frac{1}{x} + e^x$$

Ex: Find the equation of the tangent line to the curve $f(x) = \sqrt{x^3 + 2x}$ at $x = 4$.

Soln: $y - y_0 = m(x - x_0)$ ← Use point-slope formula.

Need: a point on the line (x_0, y_0)

Need: the slope m

Find the point: $f(4) = \sqrt{4^3 + 2\cdot 4} = \sqrt{64 + 8} = \sqrt{72}$

So use the point $(4, \sqrt{72})$.

Find the slope: $f'(x) = \frac{d}{dx}\left[\sqrt{x^3+2x}\right]$

$$= \frac{d}{dx}\left[(x^3+2x)^{1/2}\right]$$

Let $z = x^3 + 2x$

$$= \frac{d}{dz}\left[z^{1/2}\right] \cdot \frac{d}{dx}\left[x^3+2x\right]$$

$$= \frac{1}{2} z^{-1/2} \cdot (3x^2+2)$$

$$= \frac{1}{2\sqrt{z}} \cdot (3x^2+2)$$

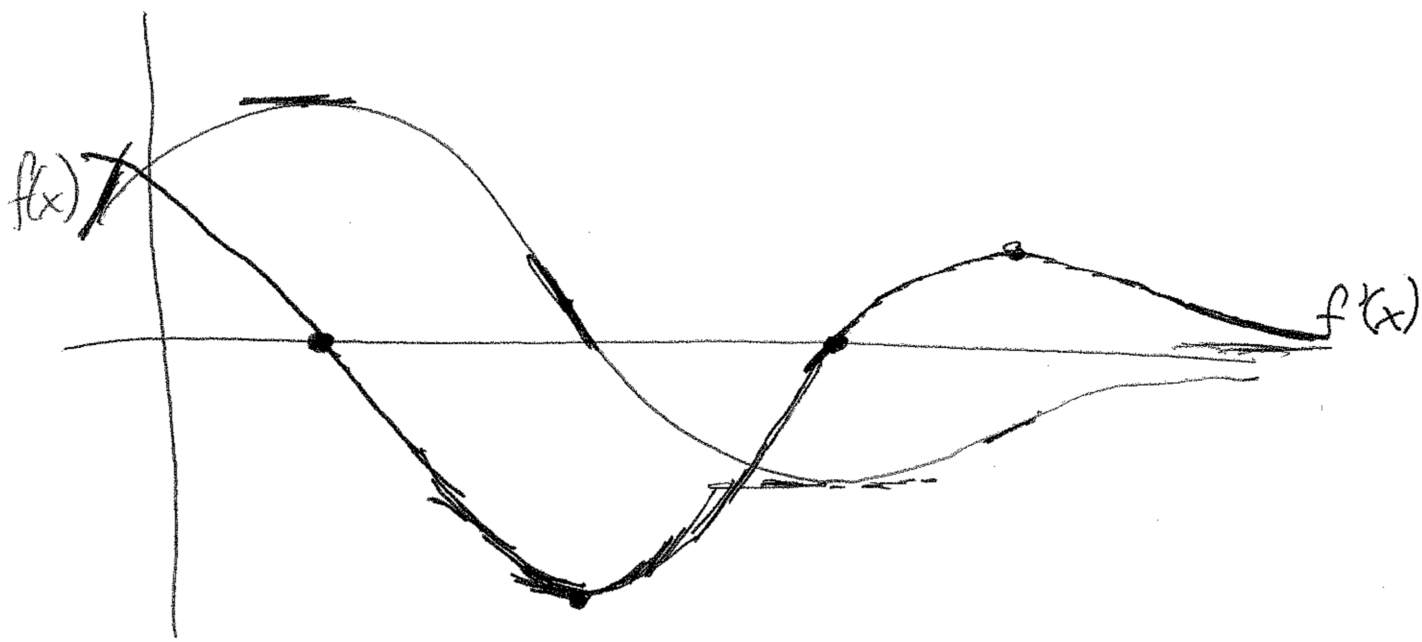
$$= \frac{3x^2+2}{2\sqrt{x^3+2x}}$$

$$m = f'(4) = \frac{3 \cdot 4^2 + 2}{2\sqrt{4^3+2\cdot 4}} = \frac{48+2}{2\sqrt{72}} = \frac{\cancel{50}\,25}{\cancel{2}\sqrt{72}} = \frac{25}{\sqrt{72}}$$

- Plug in to $y - y_0 = m(x - x_0)$

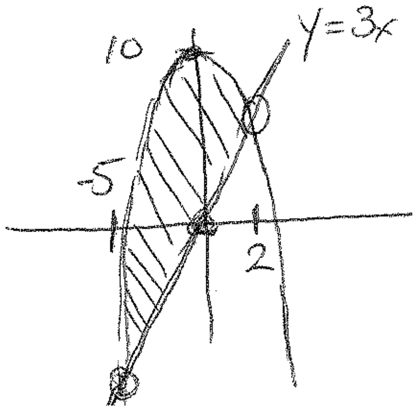
$$\boxed{y - \sqrt{72} = \frac{25}{\sqrt{72}}(x-4)}$$

Ex Given the graph of $f(x)$, sketch the derivative $f'(x)$.



Ex ~~Desc~~ Express the area of the region between the graphs of $y = 3x$ and $y = 10 - x^2$ as a definite integral.

Soln:



Find intersection pts:

$$3x = 10 - x^2$$

$$x^2 + 3x - 10 = 0$$

$$(x+5)(x-2) = 0$$

$x = -5$ or $x = 2$

$$\text{Area} = \int_{-5}^{2} (10 - x^2) - (3x)\, dx$$

$$= \boxed{\int_{-5}^{2} 10 - 3x - x^2\, dx}$$

$$= \left(10x - \frac{3x^2}{2} - \frac{x^3}{3}\right)\Big|_{-5}^{2}$$

$$= \left(10 \cdot 2 - \frac{3}{2} \cdot 2^2 - \frac{2^3}{3}\right) - \left(10(-5) - \frac{3}{2}(-5)^2 - \frac{(-5)^3}{3}\right)$$

Ex: 1.7 #16

- Prozac has a half-life of 3 days.
- What percentage of a dosage remains after 1 day?
- After 1 week?

Soln: $P = P_0 e^{kt}$ 5

$$\frac{1}{2} \cancel{P_0} = \cancel{P_0} e^{k \cdot 3}$$

$$\frac{1}{2} = e^{3k}$$

$$\ln\left(\frac{1}{2}\right) = 3k$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{3} = -0.231$$

- $P = P_0 e^{-0.231 t}$

- After 1 day, we have

$$P_0 e^{-0.231(1)} = P_0 \cdot 0.7937$$

So after 1 day, 79.37% of the dosage is left.

- After 1 week, we have

$$P_0 e^{-0.231(7)} = P_0 \cdot 0.1985$$

So after 1 week, there is 19.85% left.