Announcements

- HW 11 due Thurs 11pm
- Final Exam:

⇒ Sec 011: Fri Dec 10 9am-noon

⇒ Sec 013: SAT Dec 11 2pm-5pm

WARM-up: Find the average value of the function $f(x) = \frac{\ln x}{x}$ over $[1, e^2]$.

Soln avg val $= \frac{1}{e^2-1}\int_1^{e^2} \frac{\ln x}{x}\,dx$

- $w = \ln x$
- $\frac{dw}{dx} = \frac{1}{x}$
- $dw = \frac{1}{x}dx$

$= \frac{1}{e^2-1}\int_1^{e^2} \ln x \cdot \frac{1}{x}\,dx$

$= \frac{1}{e^2-1}\int_0^{2} w \cdot dw$

- To get new limits of integration:

$w(1) = \ln(1) = 0$

$w(e^2) = \ln(e^2) = 2\cdot\ln(e) = 2\cdot 1 = 2$

Continuing: $\frac{1}{e^2-1}\int_0^2 w\,dw$

$= \frac{1}{e^2-1}\left(\frac{w^2}{2}\right)\Big|_0^2$

$$= \frac{1}{e^2-1}\left(\frac{2^2}{2} - \frac{0^2}{2}\right)$$

$$= \boxed{\frac{2}{e^2-1} \approx 0.313}$$

Ex: A bicycle race starts at time $t=0$ hours. At time t, the speed of the bicycle (in mph) is given by

$$t\,e^{-t^2+4}.$$

(a) Find the maximum and minimum speed of the bicycle in the interval $[0, 2]$.

(b) What is the average speed of the bike during this time?

Soln (a) $f(t) = te^{-t^2+4}$

$$f'(t) = \frac{d}{dt}\left[t \cdot e^{-t^2+4}\right]$$

$$= \frac{d}{dt}[t] \cdot e^{-t^2+4} + t \cdot \frac{d}{dt}\left[e^{-t^2+4}\right]$$

$$= 1 \cdot e^{-t^2+4} + t \cdot e^{-t^2+4} \cdot \frac{d}{dt}[-t^2+4]$$

$$= e^{-t^2+4} + t \cdot e^{-t^2+4} \cdot (-2t)$$

$$= e^{-t^2+4}(1 - 2t^2)$$

- Find crit pts:

$$e^{-t^2+4}(1 - 2t^2) = 0$$

$e^{-t^2+4} = 0$ or $1 - 2t^2 = 0$

No Soln $1 = 2t^2$

$\frac{1}{2} = t^2$

$t = \pm\sqrt{\frac{1}{2}}$

- Check $f(t)$: $f(t) = te^{-t^2+4}$
  - $t=0$: $f(0) = 0 \cdot e^4 = 0$
  - $t = \sqrt{\frac{1}{2}}$ $f(\sqrt{\frac{1}{2}}) = \sqrt{\frac{1}{2}} \cdot e^{-\frac{1}{2}+4} = 23.4$
  - $t = 2$ $f(2) = 2 \cdot e^{-4+4} = 2$

- The minimum speed of the bike is 0mph.
- The maximum speed of the bike is 23.4 mph.

(b) Find the avg ~~val~~ speed of the bike.

$$\text{Avg val} = \frac{1}{2-0}\int_0^2 te^{-t^2+4}\,dt$$

(5)

- $w = -t^2 + 4$
- $\frac{dw}{dt} = -2t$
- $dw = -2t \cdot dt$
- $-\frac{1}{2} dw = t \cdot dt$

$$= \frac{1}{2} \int_0^2 e^{-t^2+4} \cdot t \, dt$$

$$= \frac{1}{2} \int_4^0 e^w \cdot -\frac{1}{2} dw$$

$$= -\frac{1}{4} \int_4^0 e^w \, dw$$

$$= -\frac{1}{4} (e^w)\Big|_4^0$$

$$= -\frac{1}{4} (e^0 - e^4)$$

$$= -\frac{1}{4} (1 - e^4)$$

$$= -\frac{1-e^4}{4}$$

$$= \frac{e^4 - 1}{4} = \boxed{13.4 \text{ mph}}$$