Announcements

· Test 3 Stats

	011	013
Avg	43.4	40.13
Med	44.25	41.5
Std Dev	11.94	11.67

· Final Exam

011: Fri Dec 10 9am - noon
013: Sat Dec 11 2pm - 5pm

WARM-UP: Avg val $= \frac{1}{b-a}\int_a^b f(x)\,dx$

Find the average value of

$f(x) = \frac{1}{x^2}$ over $[\frac{1}{2}, 2]$.

<u>Soln</u>: Avg val $= \frac{1}{2-\frac{1}{2}} \int_{\frac{1}{2}}^{2} \frac{1}{x^2}\, dx$

$$= \frac{1}{3/2} \int_{\frac{1}{2}}^{2} x^{-2}\, dx$$

$$= \frac{2}{3} \left(\frac{x^{-1}}{-1} \right) \Bigg|_{\frac{1}{2}}^{2}$$

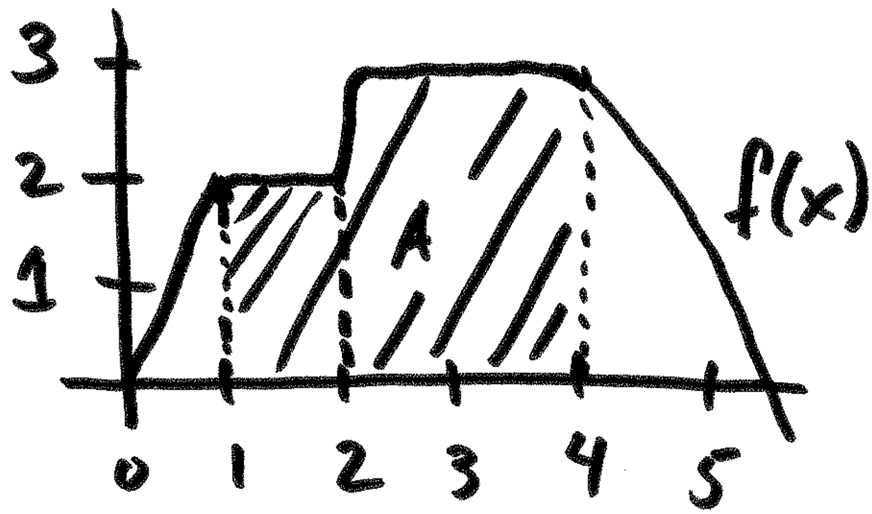
$$= \frac{2}{3} \left(-\frac{1}{x} \right) \Bigg|_{\frac{1}{2}}^{2}$$

$$= \frac{2}{3} \left(\left(-\frac{1}{2}\right) - \left(-\frac{1}{1/2}\right) \right)$$

$$= \frac{2}{3} \left(-\frac{1}{2} + 2 \right)$$

$$= \frac{2}{3} \left(\frac{3}{2} \right) = \boxed{1}$$

Ex: Estimate the average value of $f(x)$ over $[1, 4]$.



Soln: $$\text{Avg val} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{4-1}\int_1^4 f(x)\,dx$$

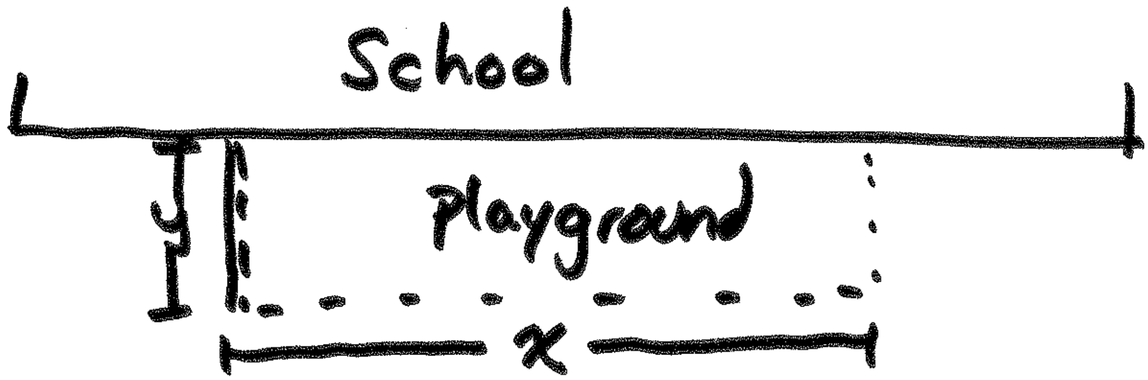
$$= \frac{1}{3}\int_1^4 f(x)\,dx$$

$$= \frac{1}{3}A =$$

$$= \frac{1}{3}(1\cdot 2 + 2\cdot 3)$$

$$= \frac{1}{3}(2+6) = \boxed{\frac{8}{3}}$$

Ex



- Playground should have an area of $72\ m^2$.
- What is the minimum amount of fencing needed?

Soln: • Area $= x \cdot y = 72$

• Fencing needed $= y + x + y = 2y + x$

• $xy = 72$

$y = \frac{72}{x}$

• Fencing needed $= 2\left(\frac{72}{x}\right) + x$

$$= \frac{144}{x} + x$$

- Need the global minimum of $f(x) = \frac{144}{x} + x$ over $(0, \infty)$

- $f(x) = 144x^{-1} + x$
- $f'(x) = -144x^{-2} + 1$
- Find crit pts:

$$-\frac{144}{x^2} + 1 = 0$$

$$1 = \frac{144}{x^2}$$

$$x^2 = 144$$

$$x = \pm 12$$

- Try $x = 12$, $x \to 0$, $x \to \infty$.

- Fencing needed: $f(x) = \frac{144}{x} + x$.

- $\underline{x \to 0}$: $f(x) \to \infty$

- $\underline{x = 12}$: $f(12) = \frac{144}{12} + 12 = 24$

- $\underline{x \to \infty}$: $f(x) \to \infty$

- So the minimum occurs when $x = 12$ (and $y = 6$), and uses $\boxed{24\ m}$ of fencing.

Ex At time t (in seconds), a fire hose delivers water at a rate of $\frac{1}{5}t^2$ gal/s.

(a) What is the average pump rate during the first 5 seconds?

(b) How long does it take to fill a 10 gal tank?

Soln: (a) Want the avg. val of $f(t) = \frac{1}{5}t^2$ over $[0, 5]$.

$$\text{Avg val} = \frac{1}{b-a}\int_a^b f(t)\,dt$$

$$= \frac{1}{5}\int_0^5 \frac{1}{5}t^2\,dt$$

$$= \frac{1}{25}\int_0^5 t^2\,dt$$

$$= \frac{1}{25}\left(\frac{t^3}{3}\right)\Big|_0^5$$

$$= \frac{1}{25}\left(\frac{5^3}{3} - \frac{0^3}{3}\right)$$

$$= \frac{1}{25}\left(\frac{25\cdot 5}{3} - 0\right)$$

$$= \frac{1}{\cancel{25}}\cdot\frac{\cancel{25}\cdot 5}{3} = \boxed{\frac{5}{3}\text{ gal/sec}}$$

(b) How long does it take to fill a 10 gal tank?

<u>Soln</u>: Let a be some amount of time.

- After a seconds, the total water delivered is

$$\int_0^a \frac{1}{5} t^2 \, dt$$

$$= \frac{1}{5} \int_0^a t^2 \, dt$$

$$= \frac{1}{5} \left(\frac{t^3}{3} \right) \Big|_0^a$$

$$= \frac{1}{5} \left(\frac{a^3}{3} - \frac{0^3}{3} \right)$$

$$= \frac{1}{5} \left(\frac{a^3}{3} \right) = \frac{a^3}{15}$$

- Solve for a in $\frac{a^3}{15} = 10$

$$a^3 = 150$$

$$(a^3)^{1/3} = (150)^{1/3} \quad (9)$$

$$a = (150)^{1/3}$$

$$\approx \boxed{5.31 \text{ seconds}}$$