

# Announcements

©

- HW 11 out today due Dec 1 (Wed)
- Feedback today

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WARM-UP: After time  $t$  (in hours),

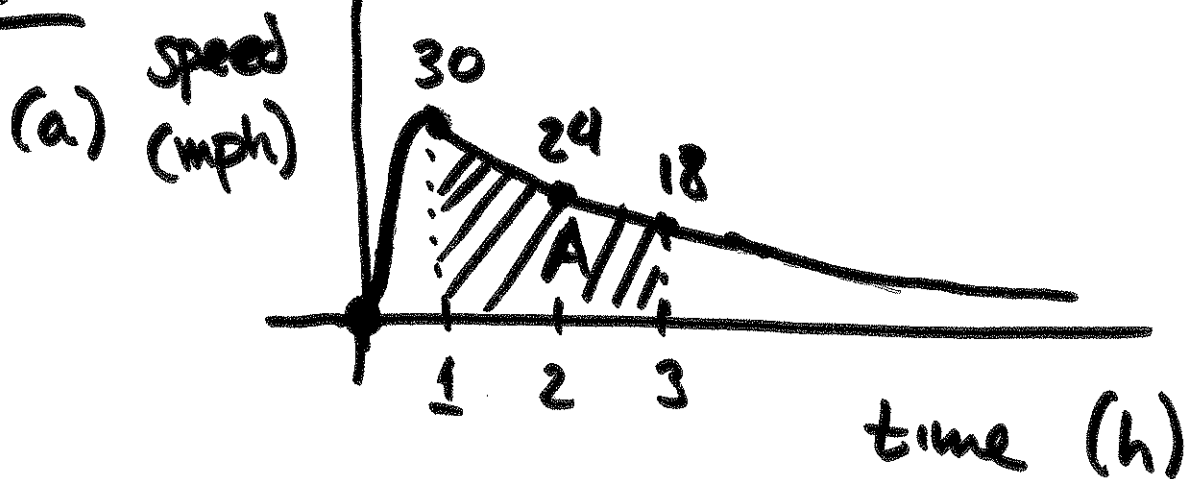
a car travels at a speed of  $\frac{60t}{1+t^2}$

miles per hour.

(a) How far did the car travel from time  $t=1$  to time  $t=3$  hours?

(b) What was the average speed of the car during this time?

(1)

Soln

$$A = \int_1^3 \frac{60t}{1+t^2} dt = \int_1^3 \frac{1}{1+t^2} \cdot 60t dt$$

$$\cdot w = 1+t^2$$

$$\cdot dw = 2t dt$$

$$\cdot 30 dw = 60t dt$$

$$= \int_2^{10} \frac{1}{w} \cdot 30 dw$$

$$= 30 \int_2^{10} \frac{1}{w} dw$$

$$= 30 \ln|w| \Big|_2^{10}$$

$$= 30 \ln(10) - 30 \ln(2)$$

$$= 30 \ln(10) - 30 \ln(2)$$

$$= 30 \ln(5)$$

$$\approx \boxed{48.28 \text{ miles}}$$

$$(b) \text{ Avg speed} = \frac{\text{total distance}}{\text{time}} \quad (2)$$

$$= \frac{48.28 \text{ miles}}{2 \text{ hrs}}$$

$$= \boxed{24.14 \text{ mph}}$$

6.1 Average value of a function

def The average value of a function  $f(x)$  over  $[a, b]$  is

given by

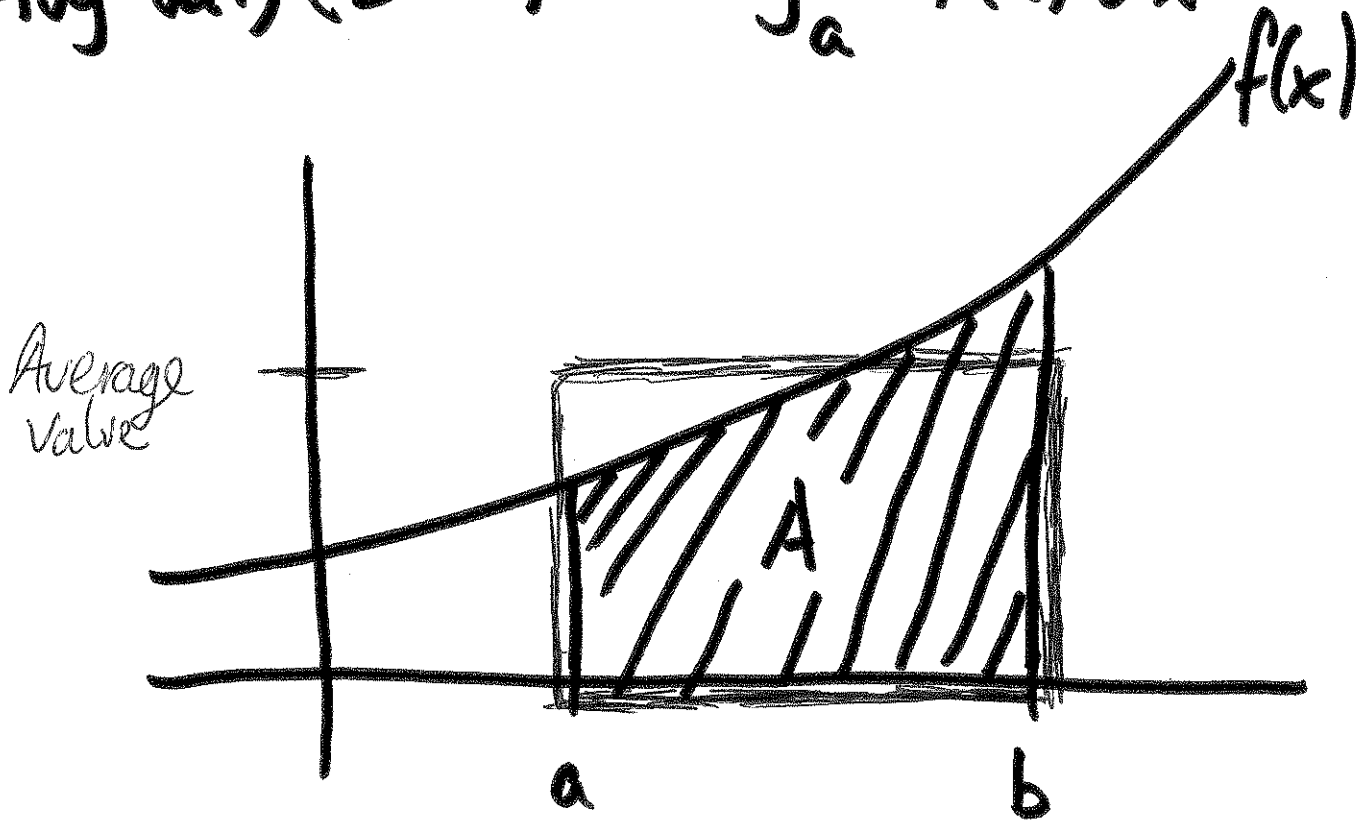
$$\boxed{\frac{1}{b-a} \int_a^b f(x) dx}$$

(3)

## Graphical Interpretation:

$$(\text{Avg val}) = \frac{1}{b-a} \int_a^b f(x) dx$$

$$(\text{Avg val})(b-a) = \int_a^b f(x) dx$$



- $A = \int_a^b f(x) dx$

- Area of Rectangle = Area under curve

(4)

Ex Find the average value  
of  $f(x) = 3x^2$  over  $[1, 5]$ .

Soln Aug val =  $\frac{1}{b-a} \int_a^b f(x) dx$

$$= \frac{1}{5-1} \int_1^5 3x^2 dx$$
$$= \frac{1}{4} x^3 \Big|_1^5$$
$$= \frac{1}{4} (5^3 - 1^3)$$
$$= \frac{1}{4} (125 - 1)$$
$$= \frac{124}{4} = \boxed{31}$$

Ex Find the avg val of  
 $f(x) = \frac{1}{\sqrt{x}}$  over  $[1, 9]$ .

⑤

Soln avg val =  $\frac{1}{b-a} \int_a^b f(x) dx$

$$= \frac{1}{9-1} \int_1^9 \frac{1}{\sqrt{x}} dx$$

$$= \frac{1}{8} \int_1^9 x^{-1/2} dx$$

$$= \frac{1}{8} (2x^{1/2}) \Big|_1^9$$

$$= \frac{1}{8} (2\sqrt{9} - 2\sqrt{1})$$

$$= \frac{1}{8} (6 - 2)$$

$$= \frac{1}{8} \cdot 4 = \boxed{\frac{1}{2}}$$

Ex Find the avg value of

(6)

$$f(x) = e^{-3x} \quad \text{over } [0, 2].$$

Soln: avg val =  $\frac{1}{b-a} \int_a^b f(x) dx$

$$= \frac{1}{2-0} \int_0^2 e^{-3x} dx$$

$$= \frac{1}{2} \left( -\frac{1}{3} e^{-3x} \right) \Big|_0^2$$

$$= \frac{1}{2} \left( \left( -\frac{1}{3} e^{-6} \right) - \left( -\frac{1}{3} e^0 \right) \right)$$

$$= \frac{1}{2} \left( -\frac{1}{3e^6} + \frac{1}{3} \right)$$

$$= \frac{1}{2} \left( -\frac{1}{3e^6} + \frac{e^6}{3e^6} \right)$$

$$= \frac{1}{2} \left( \frac{e^6 - 1}{3e^6} \right) = \boxed{\frac{e^6 - 1}{6e^6}}$$