Announcements

→ Hw 10 Due today

→ Test 3 on Friday: 4.2, 4.4, 4.5,
5.1-5.5,
7.1-7.3

Warm-up:

(a) $\int_4^9 x+1 \, dx$

(b) $\int_0^1 x^2 e^{x^3-1} \, dx$

(c) $\int_2^5 \frac{1}{x \ln x} \, dx$

Soln (a) $\int_4^9 x + 1 \, dx$

$$= \int_4^9 x \, dx + \int_4^9 1 \, dx$$

$$= \frac{x^2}{2}\Big|_4^9 + x\Big|_4^9$$

$$= \left(\frac{9^2}{2} - \frac{4^2}{2}\right) + (9 - 4)$$

$$= \frac{81}{2} - \frac{16}{2} + 5$$

$$= 40.5 - 8 + 5$$

$$= \boxed{37.5}$$

(b) $\int_0^1 x^2 e^{x^3-1} \, dx = \int_0^1 e^{x^3-1} \cdot x^2 dx$

- $w = x^3 - 1$
- $\frac{dw}{dx} = 3x^2$

$$= \int_{-1}^0 e^w \cdot \frac{1}{3} \cdot dw$$

(2)

- $\frac{dw}{3} = x^2\,dx$

$$= \frac{1}{3}\int_{-1}^{0} e^{w}\,dw$$

$$= \frac{1}{3}\left(e^{w}\right)\Big|_{-1}^{0}$$

$$= \frac{1}{3}\left(e^{0} - e^{-1}\right)$$

$$= \boxed{\frac{1}{3}\left(1 - \frac{1}{e}\right)}$$

(c) $\int_{2}^{5} \frac{1}{x \ln x}\,dx = \int_{2}^{5} \frac{1}{\ln x} \cdot \frac{1}{x} \cdot dx$

- $w = \ln x$
- $\frac{dw}{dx} = \frac{1}{x}$
- $dw = \frac{1}{x} \cdot dx$

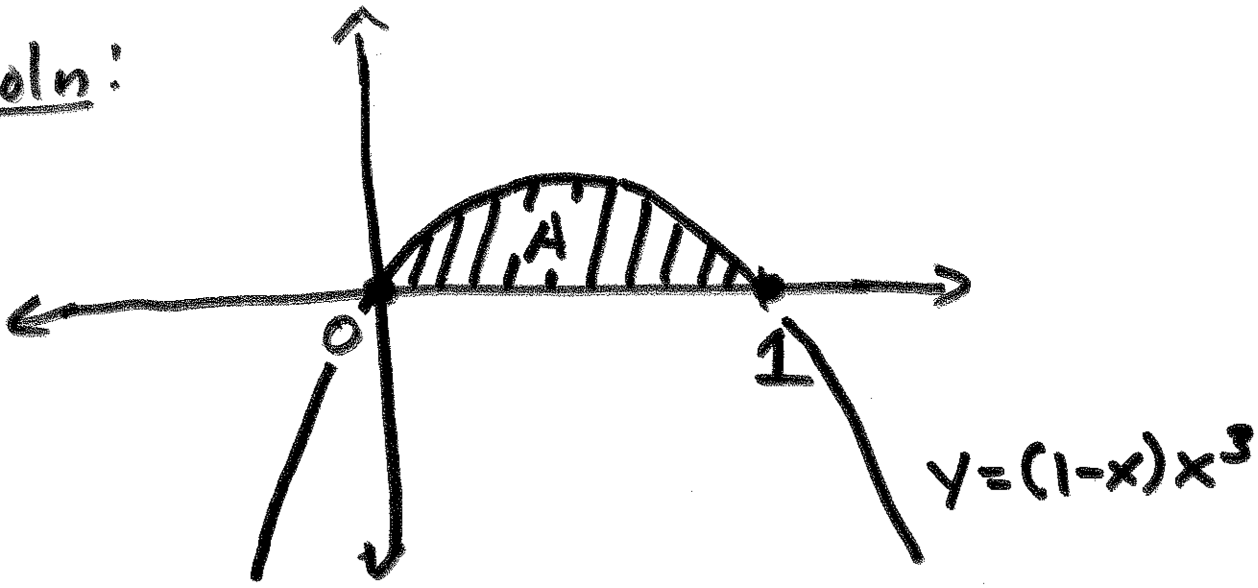
$$= \int_{\ln(2)}^{\ln(5)} \frac{1}{w} \cdot dw$$

$$= \ln|w|\Big|_{\ln(2)}^{\ln(5)}$$

$$= \boxed{\ln|\ln(5)| - \ln|\ln(2)|}$$

Ex: Find the area below the graph of $y = (1-x)\cdot x^3$ and above the x-axis.

Soln:



$$\bullet\ A = \int_0^1 (1-x)x^3\,dx$$

$$= \int_0^1 x^3 - x^4\,dx$$

$$= \int_0^1 x^3\,dx - \int_0^1 x^4\,dx$$

$$= \left.\frac{x^4}{4}\right|_0^1 - \left.\frac{x^5}{5}\right|_0^1$$

④

$$= \left(\frac{1^4}{4} - \frac{0^4}{4}\right) - \left(\frac{1^5}{5} - \frac{0^5}{5}\right)$$

$$= \left(\frac{1}{4} - 0\right) - \left(\frac{1}{5} - 0\right)$$

$$= \frac{1}{4} - \frac{1}{5}$$

$$= \frac{5}{20} - \frac{4}{20} = \boxed{\frac{1}{20}}.$$

Soln

~~#22~~

4.4 #22

At price of \$4.00, demand is 4000 units. For each \$0.25 decrease in price, the demand increases by 200 units. Find the price and quantity sold that maximizes revenue.

Soln Revenue = (price)$\cdot$(total sold)

- Need price-demand relationship.
- Let q = demand
- Know: (4, 4000) is on the price-demand curve.

- Know: $m = \dfrac{-200}{0.25} = -800$

- $(q - 4000) = -800(p - 4)$

- $q = -800p + 3200 + 4000$

  $q = -800p + 7200$

- $R(p) = p \cdot q$

  $= p(-800p + 7200)$

  $= -800p^2 + 7200p.$

- Find maximum of $R(p)$ over $[0, \infty]$

- $R'(p) = -1600p + 7200$

- Crit pts: $-1600p + 7200 = 0$

  $-1600p = -7200$

  $p = \dfrac{-7200}{-1600} = \dfrac{36}{8} = \dfrac{9}{2} = 4.5$

- Revenue is maximized when price is $4.50.

- $$q = -800(4.5) + 7200$$
$$= -3200 - 400 + 7200$$
$$= 4000 - 400 = \boxed{3600 \text{ units}}.$$

Ex $\int x^5\sqrt{3x^6+4}\,dx = \int \sqrt{3x^6+4}\;x^5\,dx$

- $w = 3x^6+4$
- $\dfrac{dw}{dx} = 18x^5$
- $\dfrac{1}{18}\cdot dw = x^5\cdot dx$

$$= \int \sqrt{w}\cdot \frac{1}{18}\,dw$$

$$= \frac{1}{18}\int \sqrt{w}\,dw$$

$$= \frac{1}{18}\int w^{1/2}\,dw$$

$$= \frac{1}{18}\left\{\frac{w^{3/2}}{3/2} + C\right.$$

$$= \frac{1}{\cancel{18}\,9}\cdot\frac{\cancel{2}}{3}\cdot w^{3/2} + C$$

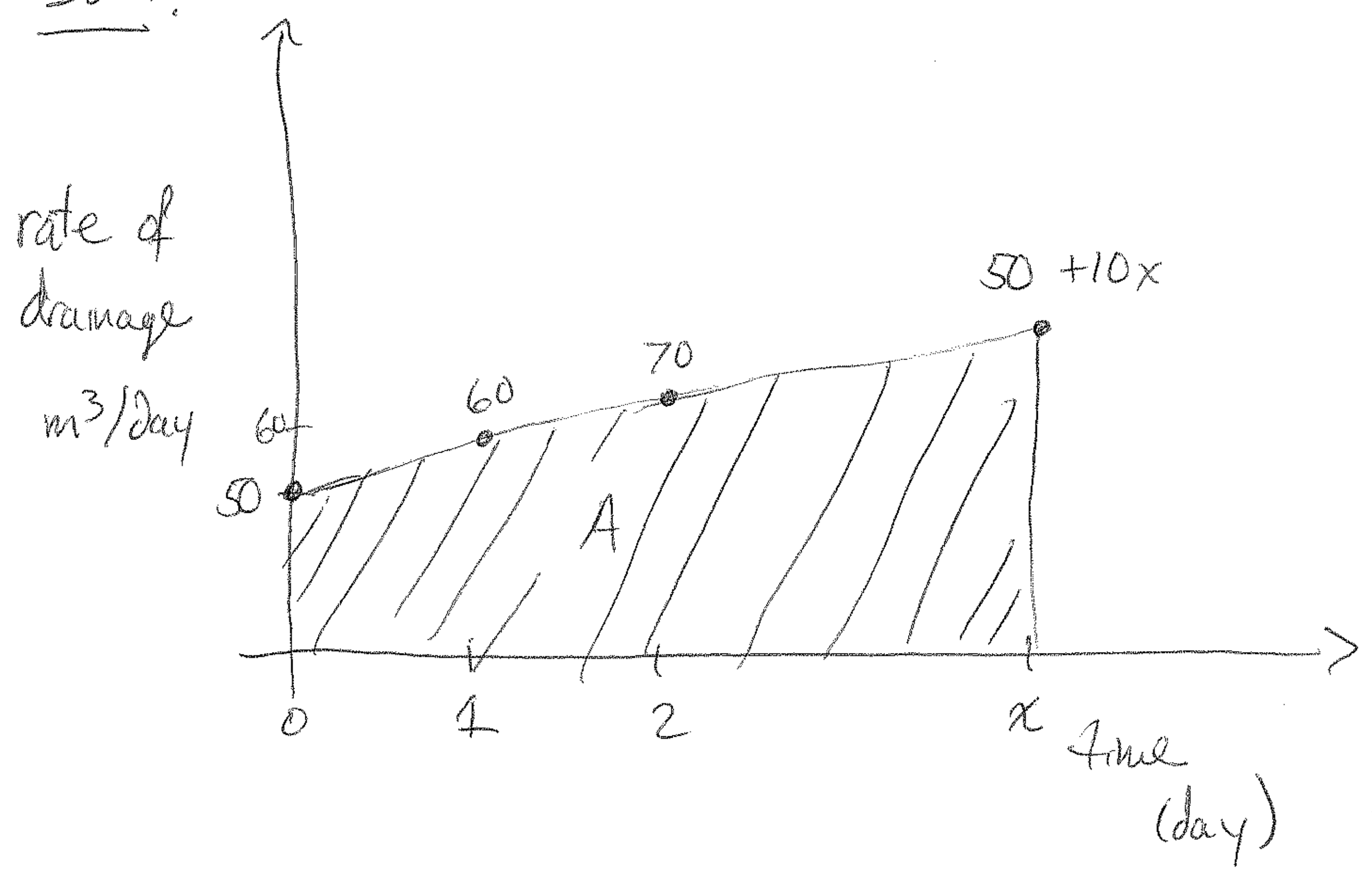
$$= \boxed{\frac{1}{27}\cdot(3x^6+4)^{3/2} + C}$$

$(3x)^6 = 3^6\cdot x^6 = 729x^6$

$3x^6$

- At time $t=0$, a water reservoir holds $200\ m^3$, and is draining at a rate of $50\ m^3/\text{day}$. With each passing day, the rate of drainage increases by $10\ m^3/\text{day}$. When will the reservoir be empty?

Soln:



- A = total amount drained at time $t=x$.

$$= \frac{x}{2}\cdot\left(50+(50+10x)\right)$$

$$= \frac{x}{2}(100 + 10x)$$

$$= 50x + 5x^2$$

- Solve for x in $50x + 5x^2 = 200$

$$5x^2 + 50x - 200 = 0$$

$$x^2 + 10x - 40 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{100 - 4(1)(-40)}}{2}$$

$$= \frac{-10 \pm \sqrt{260}}{2}$$

- So $x = -13.06$

$\boxed{x = 3.062}$

- So after $\boxed{\text{3.06 days}}$ the reservoir is empty.