Announcements

- HW 10 due Wednesday
- Wed OH: 3:15pm - 4:15pm
- Review Test 3 Wed
- TEST 3 Fri: 4.2, 4.4, 4.5, 5.1-5.5, 7.1-7.3

WARM-UP: Find the following.

(a) $\int 4x^2 \, dx$

(b) $\int 3e^{4t} \, dt$

(c) $\int (2z^3 + 4)^3 (6z^2) \, dz$

Soln

(a) $\int 4x^2\,dx = 4\int x^2\,dx$

$= \boxed{4\,\frac{x^3}{3} + C}$

(b) $\int 3e^{4t}\,dt = 3\int e^{4t}\,dt$

$= 3\left(\frac{1}{4}e^{4t}\right) + C$

$= \frac{3}{4}e^{4t} + C$

(c) $\int (2z^3+4)^3 \cdot 6z^2\,dz$

$w = 2z^3 + 4$

$\frac{dw}{dz} = 6z^2$

$dw = 6z^2\,dz$

$\int w^3\,dw = \frac{w^4}{4} + C$

$= \boxed{\frac{(2z^3+4)^4}{4} + C}$

Ex: $\int (x^3 + 2x^2)^8 \cdot (3x^2 + 4x)\, dx$

Soln: $w = x^3 + 2x^2$

$$\cancel{dx}\,\frac{dw}{\cancel{dx}} = (3x^2 + 4x)\, dx$$

$$\int w^8\, dw = \frac{w^9}{9} + C$$

$$= \frac{(x^3 + 2x^2)^9}{9} + C$$

Ex: $\int \frac{(\ln t)^5}{t}\, dt = \int (\ln t)^5 \cdot \frac{1}{t}\, dt$

- $w = \ln t$
- $\frac{dw}{dt} = \frac{1}{t}$
- $dw = \frac{1}{t}\, dt$

$$= \int w^5\, dw$$

$$= \frac{w^6}{6} + C$$

$$= \boxed{\frac{(\ln t)^6}{6} + C}$$

Ex: $\int x e^{x^2}\, dx = \int e^{x^2} \cdot x\, dx$

- $w = x^2$
- $\frac{dw}{dx} = 2x$
- $dw = 2x \cdot dx$
- $\frac{1}{2} dw = x \cdot dx$

$= \int e^{w} \cdot \frac{1}{2} dw$

$= \frac{1}{2} \int e^{w}\, dw$

$= \frac{1}{2} e^{w} + C$

$= \boxed{\frac{1}{2} e^{x^2} + C}$

Ex: $\int \frac{2e^t + 10}{e^t + 5t}\, dt = \int \frac{2(e^t + 5)}{e^t + 5t}\, dt$

- $w = e^t + 5t$
- $\frac{dw}{dt} = e^t + 5$
- $dw = (e^t + 5)\, dt$

$= \int \frac{2}{e^t + 5t} \cdot (e^t + 5)\, dt$

$= \int \frac{2}{w} \cdot dw$

$= 2 \int \frac{1}{w} \cdot dw$

$= 2 \ln|w| + C$

$= \boxed{2 \ln|e^t + 5t| + C}$

7.3 Definite Integrals via FTC

FTC: If $F'(t)$ is continuous, then

$$\int_a^b F'(t)\,dt = F(b) - F(a)$$

$$= F(x)\Big|_a^b$$

- To solve $\int_a^b f(x)\,dx$:

                                    F

1. Find an anti-derivative for f. (i.e. solve $\int f(x)\,dx$)
2. $\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b$

Ex: $$\int_2^3 x^2\,dx = \frac{x^3}{3}\Big|_2^3$$

$$= \frac{(3)^3}{3} - \frac{(2)^3}{3}$$

$$= \frac{27}{3} - \frac{8}{3} = \boxed{\frac{19}{3}}$$

$\underline{Ex}$ $\int_0^2 e^t\,dt = e^t\Big|_0^2$

$= e^2 - e^0$

$= \boxed{e^2 - 1}$

$\underline{Ex}$ $\int_1^4 \frac{1}{x}\,dx = \ln|x|\Big|_1^4$

$= \ln|4| - \ln|1|$

$= \ln(4) - \ln(1)$

$= \ln(4) - 0 = \boxed{\ln(4)}$

$\underline{Ex}$ $\int_4^{16} \frac{1}{\sqrt{x}}\,dx = \int_4^{16} \frac{1}{x^{1/2}}\,dx$

$= \int_4^{16} x^{-1/2}\,dx$

$= \frac{x^{1/2}}{1/2}\Big|_4^{16}$

$= 2\sqrt{x}\,\Big|_4^{16}$

$$= 2\sqrt{16} - 2\sqrt{4}$$

$$= 2 \cdot 4 - 2 \cdot 2$$

$$= 8 - 4 = \boxed{4}$$

Ex $\int_1^3 2x(x^2+1)^3 \, dx = \int_1^3 (x^2+1)^3 \cdot 2x \, dx$

- $w = x^2+1$
- $\frac{dw}{dx} = 2x$
- $dw = 2x \, dx$

$$= \int_{w(1)}^{w(3)} w^3 \, dw$$

$$= \int_2^{10} w^3 \, dw$$

$$= \left. \frac{w^4}{4} \right|_2^{10}$$

$$= \frac{10^4}{4} - \frac{2^4}{4}$$

$$= \frac{100 \cdot 100}{4} - \frac{16}{4}$$

$$= 2500 - 4 = \boxed{2496}$$

Ex $\int_0^3 \frac{1}{\sqrt{t+1}}\,dt = \int_1^4 \frac{1}{\sqrt{w}}\,dw$

$w = t+1$

$\frac{dw}{dt} = 1$

$dw = dt$

$$= \int_1^4 w^{-1/2}\,dw$$

$$= 2w^{1/2}\Big|_1^4$$

$$= 2\sqrt{w}\Big|_1^4$$

$$= 2\sqrt{4} - 2\sqrt{1}$$

$$= 4 - 2 = \boxed{2}$$

Bonus: $\int 3x(x^3+2)^2\,dx$

- Try $w = x^3+2$.

  $\frac{dw}{dx} = 3x^2$

- Since $\cancel{3x^2}$ a const. mult. does not appear we can't complete the substitution.