Announcements

- Quiz 9 today
- HW 10 posted; due Wed Nov 17
- Test 3 Fri Nov 19

- $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

Ex: $\int e^{\frac{x}{2}}\,dx = \frac{1}{\frac{1}{2}}e^{\frac{1}{2}x} + C$

$= \boxed{2e^{\frac{x}{2}} + C}$

Ex $\int e^{-3x}\,dx = \frac{1}{-3}e^{-3x} + C$

$= \boxed{-\frac{1}{3}e^{-3x} + C}$

Ex: $\int e^{-3}\,dx = e^{-3}\cdot x + C$

By const. rule of integration

Ex $\int (e^x + 1)^2 \, dx$

$= \int (e^x + 1)(e^x + 1) \, dx$

$= \int (e^x)\cdot(e^x) + 2e^x + 1 \, dx$

$= \int e^{2x} + 2e^x + 1 \, dx$

$= \int e^{2x} dx + 2\int e^x dx + \int 1 dx$

$= \boxed{\frac{1}{2}e^{2x} + 2e^x + x + C}$

7.2 Integration by Substitution

Ex: $f(x) = e^{x^2}$

$f'(x) = \frac{d}{dx}\left[e^{x^2}\right]$

$= e^{x^2} \cdot \frac{d}{dx}\left[x^2\right]$

$= e^{x^2} \cdot 2x$

- So $\int 2xe^{x^2}\,dx = e^{x^2} + C$

Ex $f(x) = \ln(x^4+1)$

$$f'(x) = \frac{1}{x^4+1} \cdot \frac{d}{dx}[x^4+1]$$

$$= \frac{1}{x^4+1} \cdot (4x^3)$$

$$= \frac{4x^3}{x^4+1}$$

So $\int \frac{4x^3}{x^4+1}\,dx = \ln(x^4+1) + C$

- How do we reverse this process?

Ex $\int 3x^2\sqrt{x^3+2}\,dx$

- Look for an inner function.
- The derivative of the inner function should be present.

Try $w = x^3 + 2$.

$$\frac{dw}{dx} = 3x^2$$

$$dw = 3x^2 \cdot dx$$

$$\int 3x^2 \sqrt{x^3+2}\, dx$$

$$= \int \sqrt{x^3+2} \cdot 3x^2 dx$$

$$= \int \sqrt{w} \cdot dw$$

$$= \int w^{1/2}\, dw$$

$$= \frac{w^{3/2}}{3/2} + C$$

$$= \frac{2}{3} w^{3/2} + C$$

$$= \boxed{\frac{2}{3}(x^3+2)^{3/2} + C}$$

Ex $\int 4x^3 e^{x^4}\,dx$

ILLEGAL! $\left(\int 4x^3\,dx\right)\cdot\left(\int e^{x^4}\,dx\right)$

- $w = x^4$

$\frac{dw}{dx} = 4x^3$

$dw = 4x^3\,dx$

$$\int 4x^3 e^{x^4}\,dx = \int e^{x^4}\cdot \underline{4x^3\,dx}$$

$$= \int e^w \cdot dw$$

$$= e^w + C$$

$$= \boxed{e^{x^4} + C}$$