Announcements

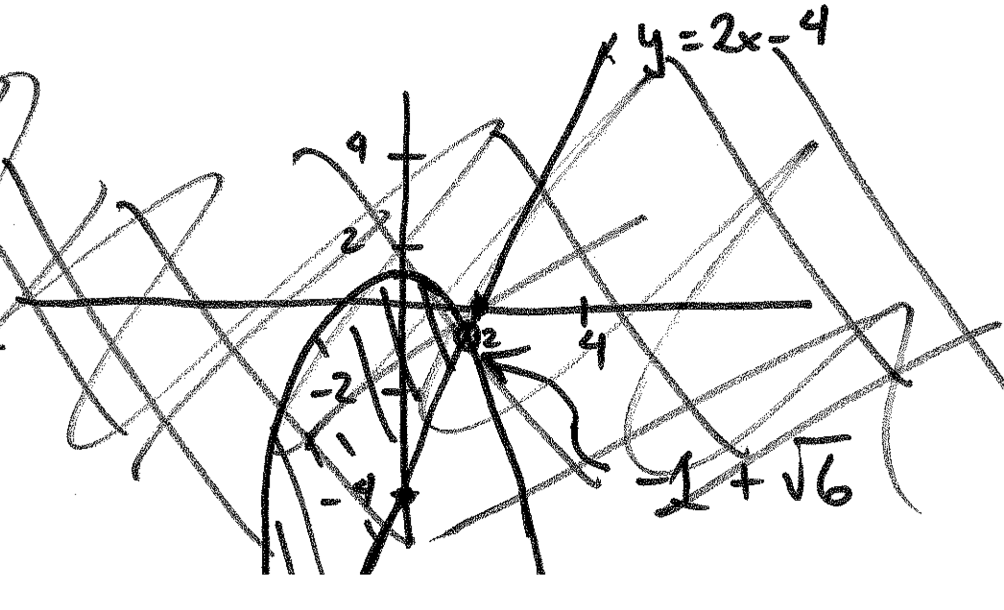
- HW 9 Due Wed
- Quiz 9 Friday <u>in</u> <u>class</u>

WARM-UP: Express the area of the region bounde ~~between~~ by the graphs of $y = 2x - 4$ and $y = 1 - x^2$ as a definite integral.

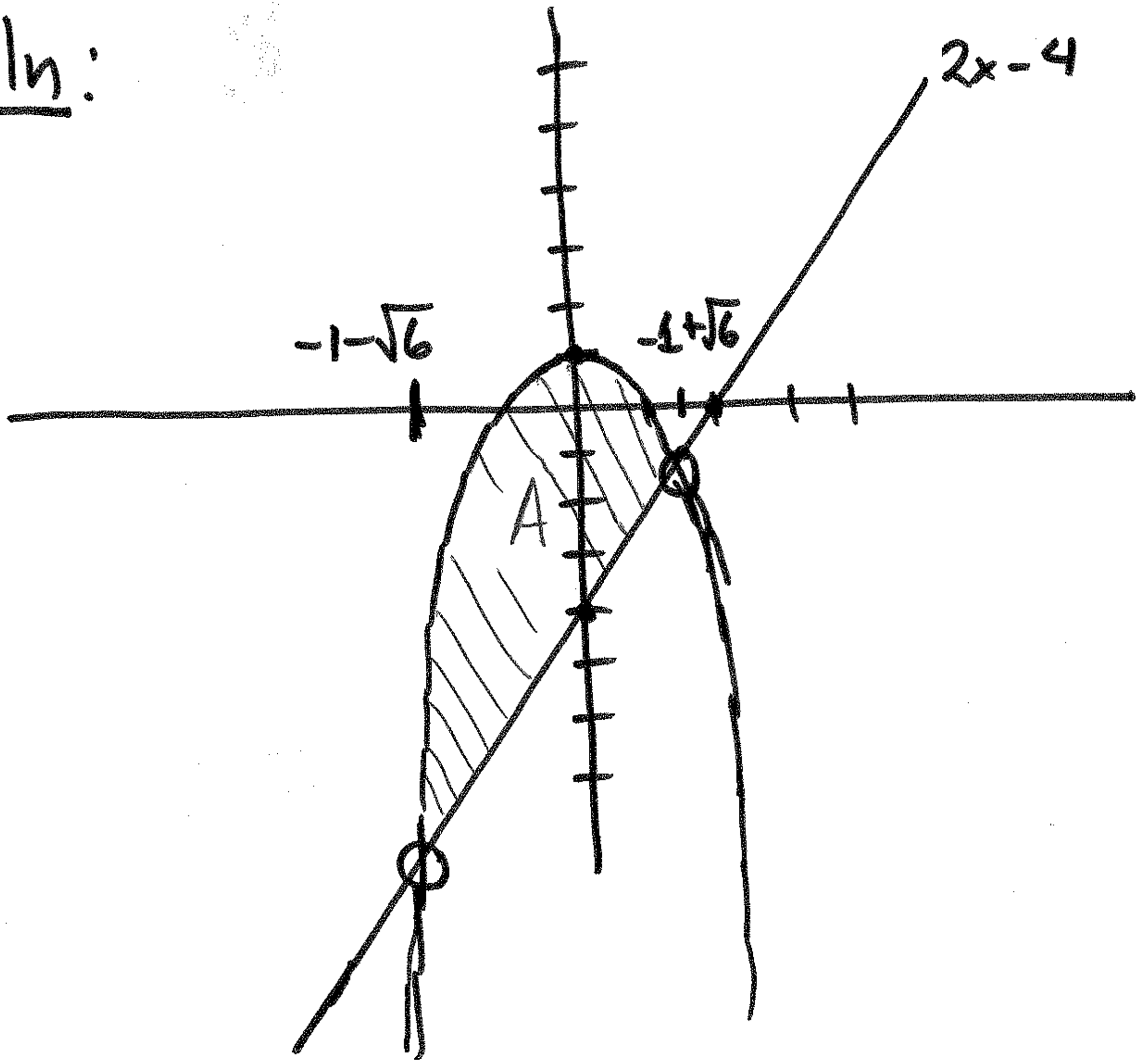
Hint: Remember the quad. formula:

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Soln:



Soln:



• Solve for x in $2x-4 = 1-x^2$

$$x^2+2x-5=0$$

$$x = \frac{-2 \pm \sqrt{4-(-20)}}{2}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 \pm \sqrt{4 \cdot 6}}{2}$$

$$= \frac{-2 \pm 2\sqrt{6}}{2}$$

$$= \frac{\cancel{2}(-1 \pm \sqrt{6})}{\cancel{2}}$$

$$= -1 \pm \sqrt{6}$$

- Area $= \boxed{\int_{-1-\sqrt{6}}^{-1+\sqrt{6}} (1-x^2) - (2x-4)\, dx}$

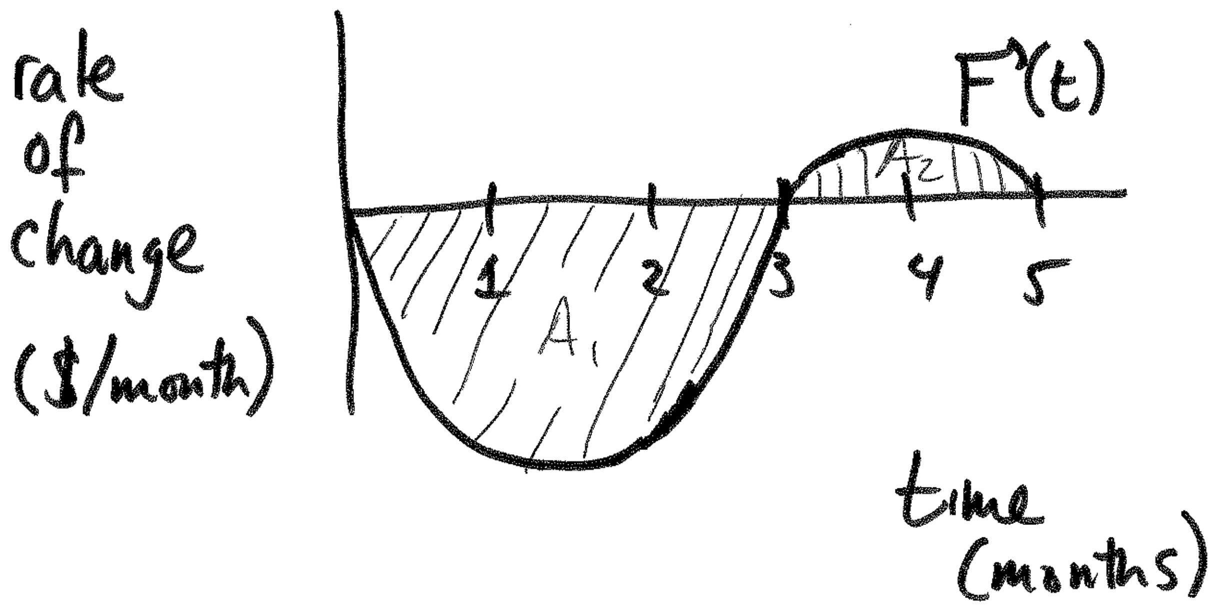
5.5 Fundamental Theorem of Calculus

If $F'(t)$ is continuous for t in $[a,b]$

then

$$\underbrace{\int_a^b F'(t)\,dt}_{\text{def. integral of the rate of change}} = \underbrace{F(b) - F(a)}_{\text{total change in function}}.$$

Ex Let $F(t)$ be the value of an investment at time t. Suppose the graph of $F'(t)$ is given below:



(a) When is the investment increasing in value?

From $t=3$ to $t=5$

(b) Overall, did the value of the investment increase or decrease?

- Is the total change $F(5)-F(0)$ positive or negative?

- By FTC, $F(5) - F(0) = \int_0^5 F'(t)\,dt$

$$= \cancel{-A_1 + A_2}$$

$$= A_2 - A_1$$

- Since $A_1 > A_2$, we get $A_2 - A_1 < 0$, and so the total change is negative and the investment lost value.

- <u>Review</u>: Differentiate $F(t) = 60[t - \ln(1+t)]$

<u>Soln</u>: $F'(t) = \frac{d}{dt}\left[60(t - \ln(1+t))\right]$

$$= 60\,\frac{d}{dt}\left[t - \ln(1+t)\right]$$

$$= 60\left(\frac{d}{dt}[t] - \frac{d}{dt}[\ln(1+t)]\right)$$

$$= 60\left(1 - \frac{1}{(1+t)}\cdot\frac{d}{dt}[1+t]\right)$$

$$= 60\left(1 - \frac{1}{1+t}\cdot 1\right)$$

$$= 60\left(\frac{1+t}{1+t} - \frac{1}{1+t}\right)$$

$$= 60\left(\frac{1+t \;\; -1}{1+t}\right)$$

$$= \boxed{60\left(\frac{t}{1+t}\right)}$$

- So if $F(t) = 60[t - \ln(1+t)]$, then

$$F'(t) = 60\left(\frac{t}{1+t}\right)$$

- By FTC,

$$\int_0^4 60\left(\frac{t}{1+t}\right) dt = F(4) - F(0)$$

$$= 60(4 - \ln(5)) -$$
$$60(0 - \ln(1))$$

$$= 60(4 - \ln(5)) \approx 143.43$$

7.1

Given an integrand, such as $60 \frac{t}{1+t}$, we want to find a function $F(t)$ so that differentiating $F(t)$ yields the integrand.

- Such a function is called an <u>antiderivative</u>.
- In other words, if $F(t)$ has the property that $F'(t) = f(t)$, then $F(t)$ is an <u>antiderivative</u> of $f(t)$.

<u>Ex</u> Let $f(t) = 4$. So $F(t) = 4t$ is an antiderivative.

✓ $F(t) = 4t + 8$

$F'(t) = 4 + 0$
$= 4$

$\cancel{F(t) = 2t^2}$

$\cancel{F'(t) = 4t}$

So $F(t) = 2t^2$ is <u>not</u> an antiderivative of $f(t) = 4$, but $F(t) = 4t + 8$ is.

- In fact, for any constant C, $F(t) = 4t + C$ is an antiderivative for $f(t) = 4$.

- <u>In</u> fact, these functions are the <u>only</u> antiderivatives for $f(t) = 4$.

- We say that $4t + C$ is the <u>family of antiderivatives</u> of the function $f(t) = 4$.