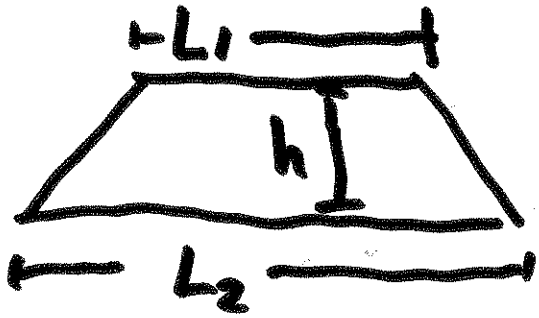
Announcements

- Quiz 8 due Tues
- HW9 out today; due Fri

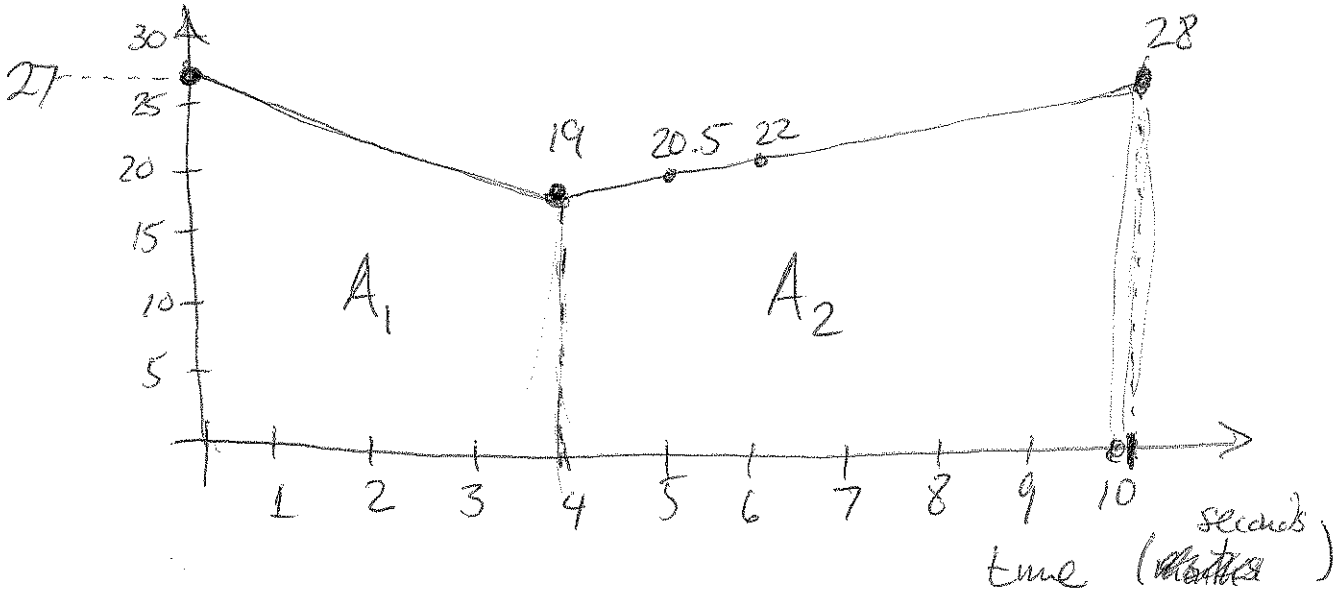
Warm-up: After t seconds of traveling uphill, a car's speed in m/s is $27 - 2t$. After 4 seconds, the car begins traveling downhill and its speed increases by 1.5 m/s each second. How far does the car travel between $t = 0$ seconds and $t = 10$ seconds?

Hint:



Area $= \frac{h}{2}(L_1 + L_2)$

Soln



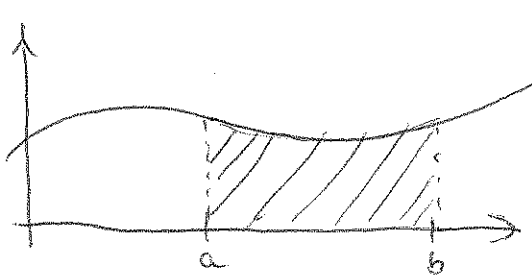
$$\text{Distance} = \text{Area} = A_1 + A_2$$

$$= \frac{4}{2}(27+19) + \frac{6}{2}(19+28)$$

$$= \boxed{233 \text{ meters}}$$

5.3: Definite Integral as Area

- So far, in our definite integrals $\left(\int_a^b f(x)\,dx\right)$, the integrand $f(x)$ is at least 0 between for x between a and b.



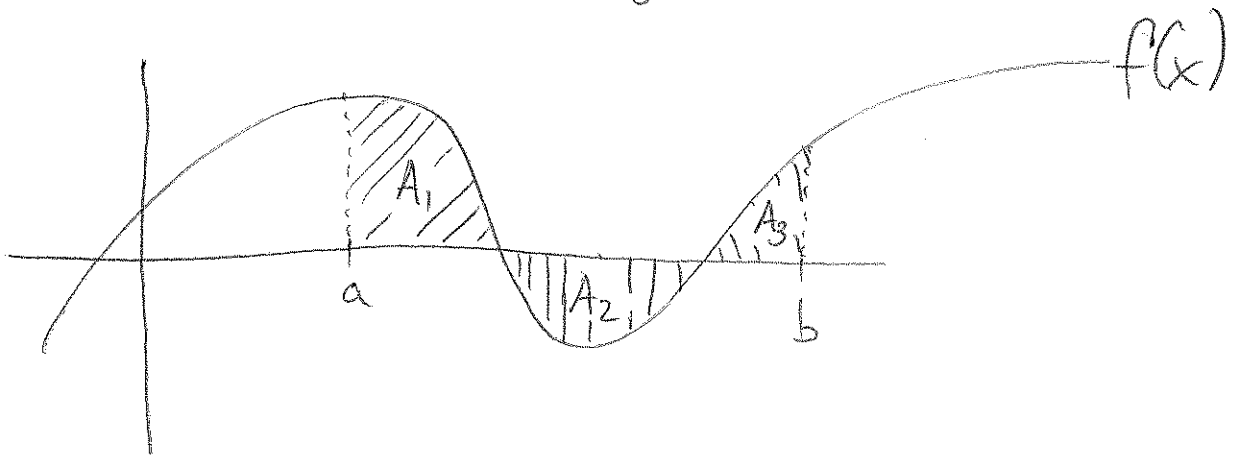
- $\int_a^b f(x)\,dx$ = Area of

- What if $f(x) < 0$? In this case, $\int_a^b f(x)\,dx$ counts ~~are~~ the "signed" area between the ~~curve~~ graph of f and the horizontal axis.

  $\Rightarrow$ Regions above the horizontal axis are counted positively

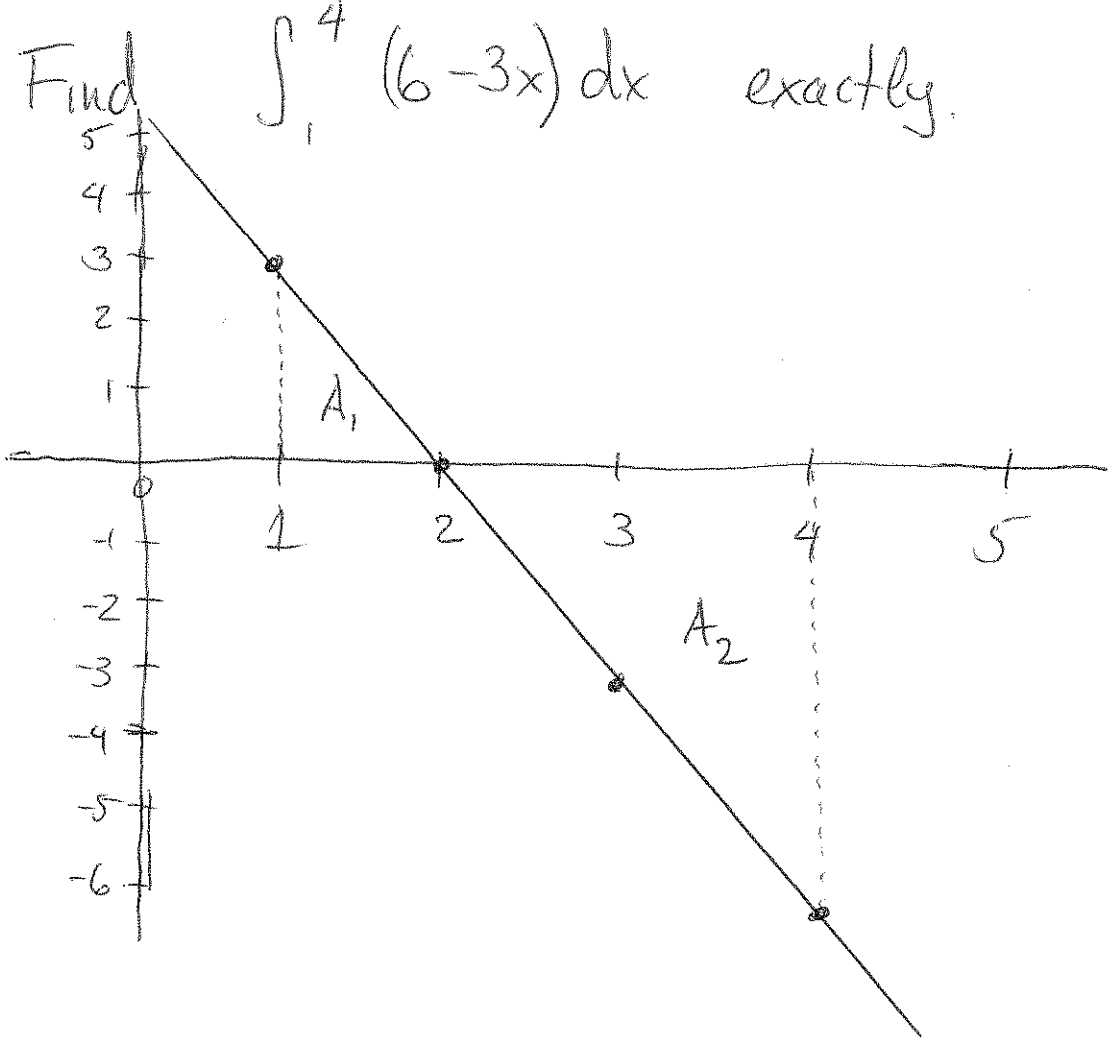
  $\Rightarrow$ Regions below the horizontal axis are counted negatively

Ex:



$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$

Ex Find $\int_1^4 (6-3x)\,dx$ exactly.



Area $= \frac{1}{2}bh$

Soln:

$$\int_1^4 (6-3x)\,dx = A_1 - A_2$$

$$= \tfrac{1}{2}\cdot 1\cdot 3 - \tfrac{1}{2}\cdot 2\cdot 6$$

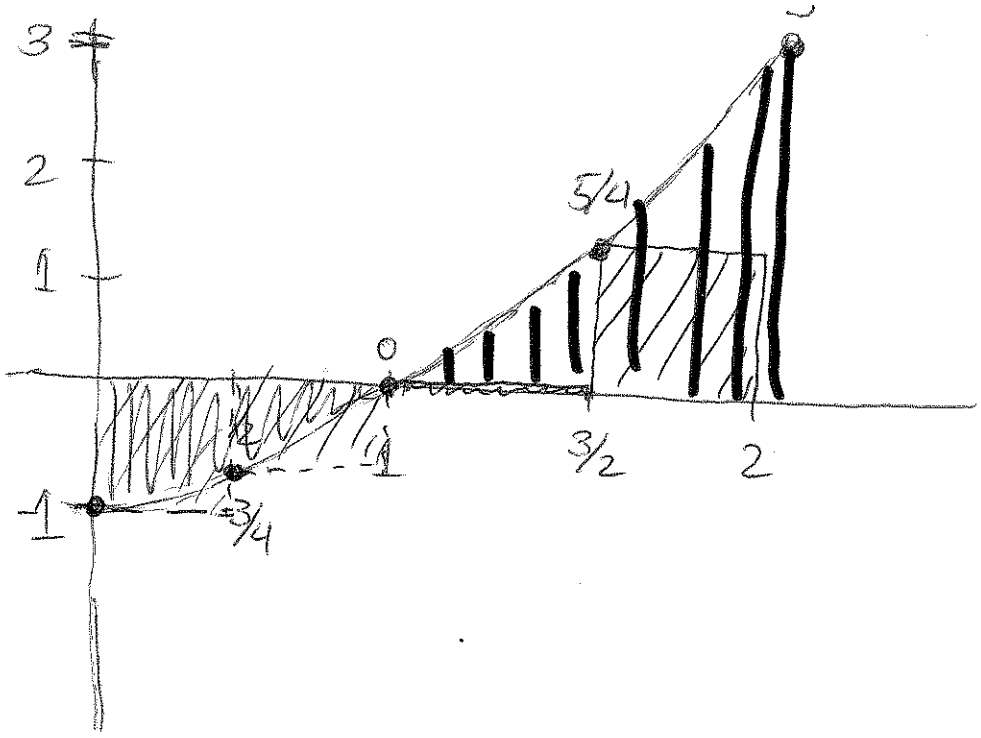
$$= \frac{3}{2} - 6 = \frac{3}{2} - \frac{12}{2} = \boxed{\frac{-9}{2}}$$

Ex: Using 4 rectangles, estimate $\int_0^2 (x^2 - 1)\,dx$

with (a) the left hand sum, and

(b) the right hand sum.

Soln



n=4: ⟵ x ⟵ x ⟵ x ⟶ ~~$\frac{3}{2}$ $\frac{9}{4}$ 1 $\frac{8}{4}$~~

L.H. Sum: $(-1 \cdot \frac{1}{2}) + (-\frac{3}{4} \cdot \frac{1}{2}) + (0 \cdot \frac{1}{2}) + (\frac{5}{4} \cdot \frac{1}{2})$

$= -\frac{4}{8} - \frac{3}{8} + 0 + \frac{5}{8}$

$= -\frac{7}{8} + \frac{5}{8} = -\frac{2}{8} = \boxed{-\frac{1}{4}}$