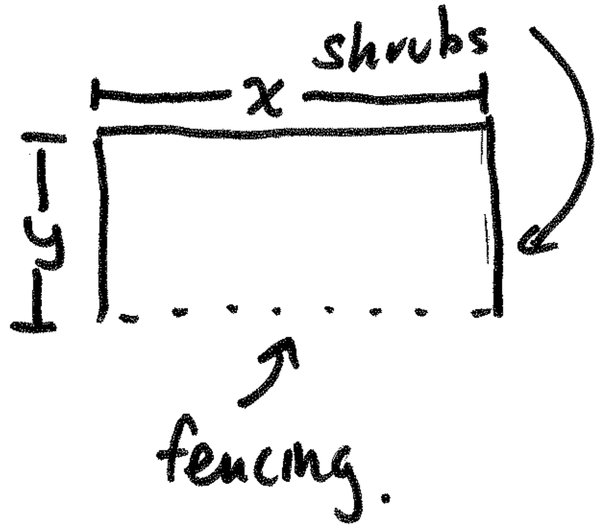
#26. A landscape architect plans to enclose a 3000 sq. ft rectangular region in a botanical garden. Three of the sides are enclosed by shrubs at a price of \$45/foot. The fourth side uses fencing at a price of \$20/foot. Find the minimum total cost.

Soln:



Know:

- $xy = 3000$
- $y = \dfrac{3000}{x}$

- $$\text{Cost} = 45(y + x + y) + 20x$$
$$= 45(2y + x) + 20x$$
$$= 45\left(2 \cdot \frac{3000}{x} + x\right) + 20x$$

$$= 45(2)\frac{3000}{x} + 45x + 20x$$

$$= 90 \cdot \frac{3000}{x} + 65x$$

$$= 9 \cdot 3 \cdot 10 \cdot 1000 \cdot \frac{1}{x} + 65x$$

$$= 270{,}000 \cdot \frac{1}{x} + 65x$$

<u>Want</u>: Global min of cost function for x in $(0, \infty)$.

$$C'(x) = \frac{d}{dx}\left[270{,}000 \cdot x^{-1} + 65x\right]$$

$$= -270{,}000 \cdot x^{-2} + 65$$

$$-270{,}000\, x^{-2} + 65 = 0$$

$$65 = 270{,}000\, x^{-2}$$

$$65 = \frac{270{,}000}{x^2}$$

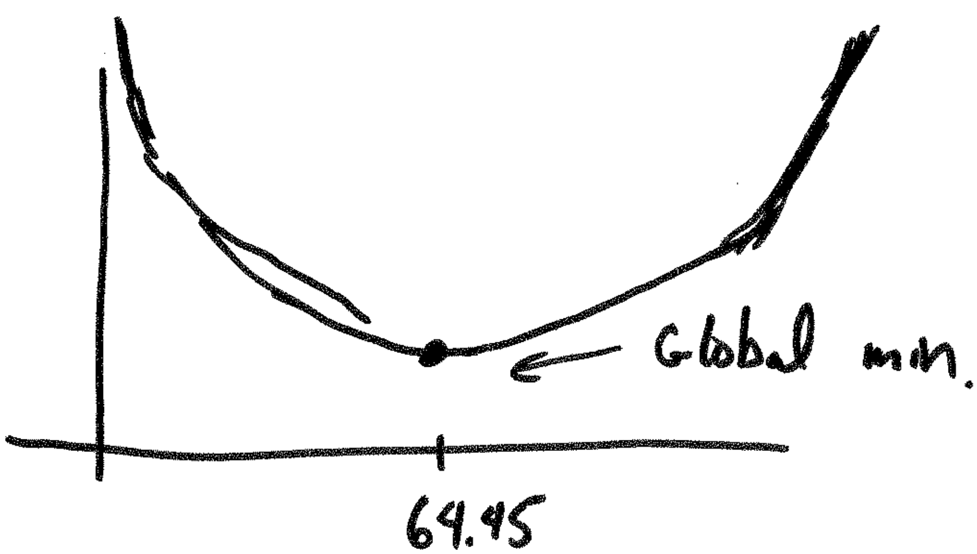
$$65x^2 = 270{,}000$$

$$x^2 = \frac{270{,}000}{65}$$

$$x = \pm\sqrt{\frac{270{,}000}{65}}$$

$$\approx \pm 64.45$$

- Recall: $C(x) = 270{,}000 \cdot \frac{1}{x} + 65x$
- Want Global min for x in $(0, \infty)$
- As $x \to \infty$, $C(x) \to \infty$
- As $x \to 0$, $C(x) \to \infty$



- Minimum cost is obtained by setting

$x = 64.45$

- $C(64.45) = \boxed{\$8378.54}$

4.4: Ex: In terms of the marginal cost and the marginal revenue, when does a company maximize profit?

Soln:
- Suppose $C(q)$ is the cost function
- Suppose $R(q)$ is the revenue function.
- Then profit function is

$$P(q) = R(q) - C(q)$$

- Want a global max of $P(q)$ for q in $[0, \infty)$

- $P'(q) = \frac{d}{dq}\left[R(q) - C(q)\right]$

$$= R'(q) - C'(q)$$

$$= MR(q) - MC(q)$$

- $MR(q) - MC(q) = 0$
- $MR(q) = MC(q)$

- So profit is maximized either when
  - $q = 0$, or
  - Marginal Revenue = Marginal Cost,

  or
  - there is no global maximum

4.5:

If $C(q)$ is the cost function, then the **average cost** $a(q)$ of producing q items is given by

$$a(q) = \frac{C(q)}{q}$$

Ex: If the cost of producing q books is $1000 + 20q$, find the average cost of producing

(a) 10 books, and

(b) $\cancel{1}00$ books.

Soln . $C(q) = 1000 + 20q$

(a) $a(10) = \frac{C(10)}{10}$

$$= \frac{1000 + 20\cdot 10}{10}$$

$$= \boxed{\$120}$$

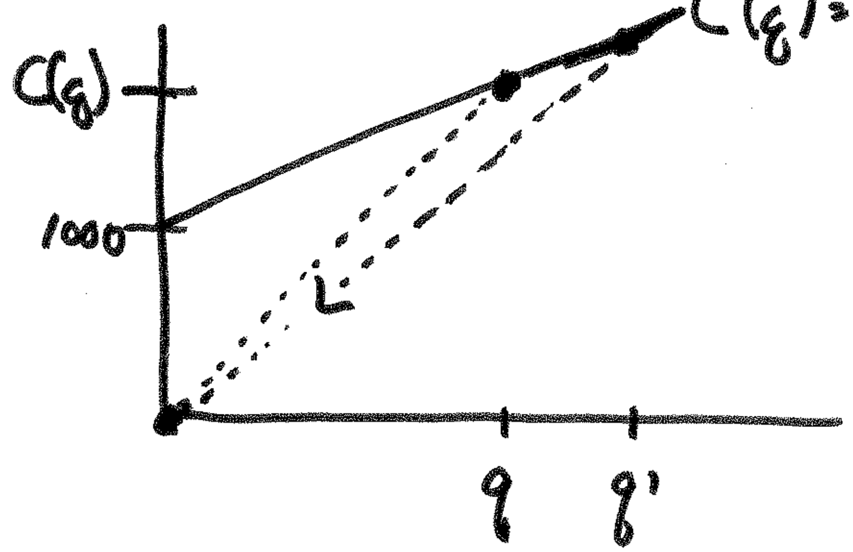
(b) $$a(100) = \frac{C(100)}{100}$$

$$= \frac{1000 + 20\cdot 100}{100}$$

$$= 10 + 20 = \boxed{\$30}$$

Average Cost Graphically:

$C(q) = 1000 + 20q$



$$a(q) = \frac{C(q)}{q} = \frac{C(q) - 0}{q - 0}$$

- $a(q)$ = slope of line L between $(0,0)$ and $(q, C(q))$

• When is Average cost minimized?

⇒ Want global min of $a(q)$ for q in $(0, \infty)$

⇒ $$a(q) = \frac{C(q)}{q}$$

$$a'(q) = \frac{q \cdot C'(q) - C(q) \cdot 1}{q^2}$$

$$\frac{q \cdot C'(q) - C(q)}{q^2} = 0$$

$$q \cdot C'(q) - C(q) = 0$$

$$q \cdot C'(q) = C(q)$$

$$C'(q) = \frac{C(q)}{q}$$

Marginal Cost = Avg Cost

- So average cost is minimized when

Marginal Cost = Avg Cost,

(or there is no global min for avg. cost.)