# Announcements

- Test 2 (2.3-2.5, 3.1-3.4, 4.1, 4.3)

  ~~M6~~ Wednesday

- Office hours today: 3:30-4:30 pm

  307 LeConte

- Today: Test 2 Review

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- Eqn of tan. line
- Interp. of derivative.
- p. 176 #33, 35: Unspecified consts in functions
- Local min/max global min, max (185 #17).
- Marginal Cost

p. 176 #33 (a)

For $a$, a positive constant, find all critical points of $f(x) = x - a\sqrt{x}$.

Ex: If $a = 2$, find the eqn. of the tangent line at $x = \cancel{3}.\ 16$

Soln: $f(x) = x - 2\sqrt{x}$

- Use point-slope to get tan. line eqn.

- Point: $\cancel{f(3) = 3 - 2\sqrt{3}}$

$$f(16) = 16 - 2\sqrt{16} = 16 - 2 \cdot 4 = 8$$

$(16, 8)$

- Slope: $f'(x) = \frac{d}{dx}\left[x - 2\sqrt{x}\right]$

$$= \frac{d}{dx}\left[x - 2x^{\frac{1}{2}}\right]$$

$$= 1 - 1x^{-\frac{1}{2}}$$

$$m = f'(16) = 1 - 1\cdot(16)^{-\frac{1}{2}}$$

$$= 1 - \frac{1}{16^{1/2}}$$

$$= 1 - \frac{1}{\sqrt{16}}$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$$y - y_0 = m(x - x_0) \qquad (x_0, y_0) = (16, 8)$$

$$m = \frac{3}{4}$$

$$\boxed{y - 8 = \frac{3}{4}(x - 16)}$$

= p.176 #33(a). Find crit. points of $f(x) = x - a\sqrt{x}$.

<u>Soln:</u> $f'(x) = \frac{d}{dx}\left[x - a\sqrt{x}\right]$

$$= \frac{d}{dx}\left[x - a x^{\frac{1}{2}}\right]$$

$$= \frac{d}{dx}[x] - a\frac{d}{dx}\left[x^{\frac{1}{2}}\right]$$

$$= 1 - a\left(\frac{1}{2}x^{-1/2}\right)$$

$$= 1 - a\left(\frac{1}{2x^{1/2}}\right)$$

$$= 1 - \frac{a}{2x^{1/2}}$$

Solve for x in $1 - \frac{a}{2x^{1/2}} = 0$

$$+\frac{a}{2x^{1/2}} = +1$$

$$a = 2x^{1/2}$$

$$\frac{a}{2} = x^{1/2}$$

$\boxed{\text{requires } a \geq 0}$

$$\left(\frac{a}{2}\right)^2 = x$$

$$x = \frac{a^2}{4}$$

So f has one critical point at $x = \frac{a^2}{4}$.

(b) What value of a gives a crit. point at $x=5$? Is the crit point a local min / local max, or neither?

<u>Soln:</u> Want: $5 = \frac{a^2}{4}$

$$20 = a^2$$

$$a = \pm\sqrt{20}$$

- Since $a \geq 0$: $a = +\sqrt{20}$.

So ~~both $a = -\sqrt{20}$ and~~ $a = \sqrt{20}$ gives a crit. point ~~s~~ at $x = 5$.

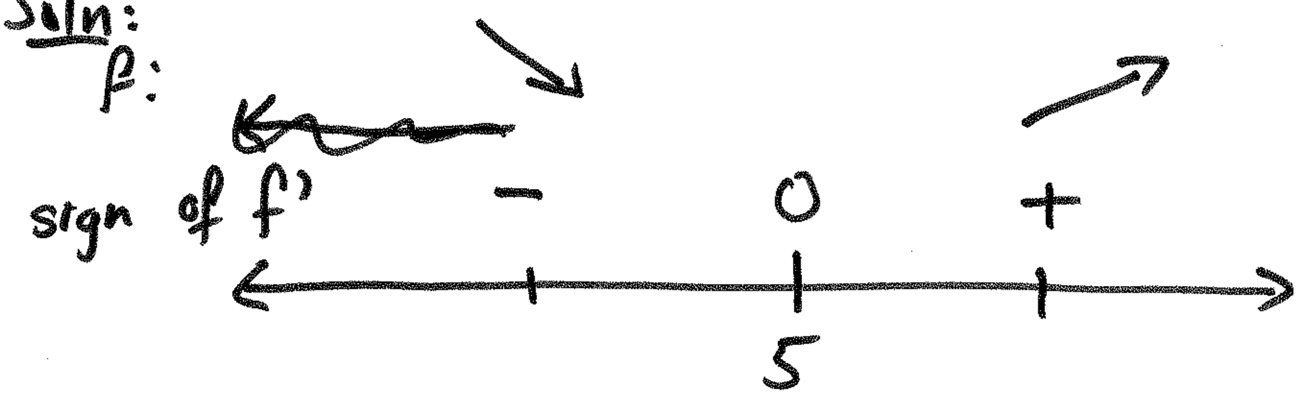
- If $a = \sqrt{20}$:

$$f(x) = x - a\sqrt{x}$$
$$= x - \sqrt{20} \cdot \sqrt{x}$$

$$f'(x) = 1 - \frac{a}{2\sqrt{x}}$$
$$= 1 - \frac{\sqrt{20}}{2\sqrt{x}}.$$

- Is $x = 5$ a local min / local max / neither?

Soln:




So at $x = 5$, we have a local minimum.

p. 187 # 38:

- $a, b$ are positive constants
- $y = ax\,e^{-bx}$ ← # of fish adults

$y$ ← # of fish offspring

(a) Find and classify the critical points.

Soln.:

$$\frac{dy}{dx} = \frac{d}{dx}\left[ax\,e^{-bx}\right]$$

$$= \frac{d}{dx}[ax]\cdot e^{-bx} + ax\cdot\frac{d}{dx}\left[e^{-bx}\right]$$

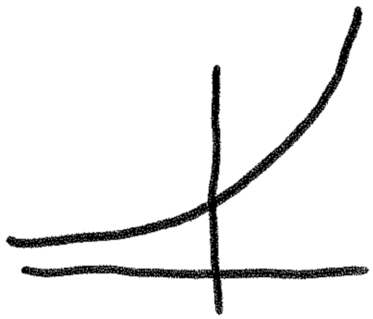
$$= a\cdot e^{-bx} + ax\cdot\left(-b\cdot e^{-bx}\right)$$

$$= ae^{-bx}\left(1 + x\cdot(-b)\right)$$

$$= ae^{-bx}(1 - bx)$$

Solve for $x$ in $\quad a e^{-bx}(1-bx)=0$

$a e^{-bx} = 0 \quad$ or $\quad 1 - bx = 0$



$1 = bx$

$x = \dfrac{1}{b}$

No Soln

· So $f$ has 1 critical point at $x = \dfrac{1}{b}$.

$f'(x) = a e^{-bx}(1-bx)$

f: ↗ ↘

sign $f'$: $\quad + \quad 0 \quad -$

$\dfrac{1}{b}$

So $f$ has a local max at $x = \dfrac{1}{b}$.

- Local extrema
- Tangent line equation
- Units of derivatives
- Product Rule / Quotient Rule
- Marginal Revenue / Marg Cost
- Interpreting Second derivative

Ex: Find the equation of the tangent line to the curve $f(x) = \frac{x^2+4}{x+\ln(x)}$ at $x = 1$.

Soln: Use point-slope formula.

- point: $f(1) = \frac{1^2+4}{1+\ln(1)} = \frac{5}{1} = 5$

$(1, 5)$.

- slope: $f'(x) = \frac{d}{dx}\left[\frac{x^2+4}{x+\ln(x)}\right]$

②

$$= \frac{(x+\ln(x))\frac{d}{dx}[x^2+4] - (x^2+4)\frac{d}{dx}[x+\ln(x)]}{(x+\ln(x))^2}$$

$$= \frac{(x+\ln(x))\cdot 2x - (x^2+4)(1+\frac{1}{x})}{(x+\ln(x))^2}$$

$$m = f'(1) = \frac{(1+\overset{0}{\ln(1)})\cdot 2 - (1^2+4)(1+\frac{1}{1})}{(1+\ln(1))^2}$$

$$= \frac{1\cdot 2 - 5\cdot 2}{1^2}$$

$$= \frac{2-10}{1} = -8$$

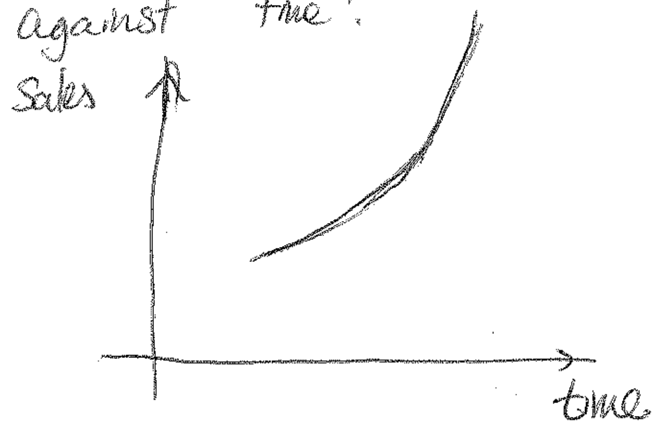
- $y - y_0 = m(x - x_0)$ where $(x_0, y_0) = (1, 5)$
  $m = -8$

$$\boxed{y - 5 = -8(x-1)}$$

$$y = -8x + 8 + 5$$

$$\boxed{y = -8x + 13}$$

2.4 #13: What does a positive second derivative mean in the graph of sales against time?

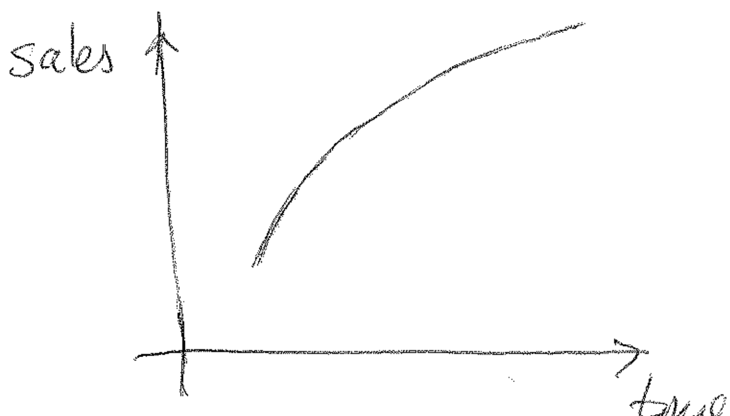



- slope is increasing
- curve is concave up

- So a positive second derivative means sales are increasing faster and faster. (I.E. The rate of increase in sales is growing.)

$\Longrightarrow$ Good advertising Campaign.

What if the second derivative is negative?




- slope is positive (all advertising helps)
- slope is decreasing
- concave down

- A negative second derivative means sales increase at a slower and slower rate.

$\Longrightarrow$ Not a great campaign.

- What are the unites of the second derivative? (Assume that sales are in <u>dollars</u> and time is measured in <u>months</u>.)

<u>Soln</u>: • First, find units of $f'(x)$:

$$f'(x) = \frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$$

difference in y

difference in x

Units are $\frac{\text{dollars}}{\text{months}}$ or dollars per month.

- Units on $f''(x)$: $\frac{\Delta \text{ output of } f'}{\Delta \text{ input of } f'}$

Units: $\frac{\text{dollars per month}}{\text{months}} = \frac{\text{dollars/months}}{\text{months}}$

$= \frac{\text{dollars}}{(\text{month})^2}$

Units of $f''(x)$: $\frac{\text{dollars}}{(\text{months})^2}$ or dollars per month$^2$

or (dollars per month) per month.

2.5 #9 Let $C(q)$ = cost function; cost in \$ to produce $q$ items.

- $C(15) = 2300$
- $\underline{C'(15)} = 108$.

$\uparrow$
Marginal cost (15) = $MC(15) = 108$

Estimate the total cost of producing 16 items and 13 items.

Soln: • $C(16) \approx C(15) + C'(15) \cdot \Delta q$

$$\approx C(15) + C'(15)(16-15)$$

$$= 2300 + 108 \cdot 1$$

$$= 2300 + 108 = \boxed{\$2408}$$

• $C(13) \approx C(15) + C'(15) \cdot \Delta q$

$$\approx 2300 + 108 \cdot (13-15)$$

$$\approx 2300 - 2 \cdot 108$$

$$\approx 2300 - 216 = \boxed{\$2084}$$

p. 187 #38 (Related)

· Find and classify the critical points of

$$f(x) = 4xe^{-2x}.$$

Soln: $$f'(x) = \frac{d}{dx}\left[(4x)(e^{-2x})\right]$$

$$= \frac{d}{dx}[4x] \cdot e^{-2x} + 4x \cdot \frac{d}{dx}\left[e^{-2x}\right]$$

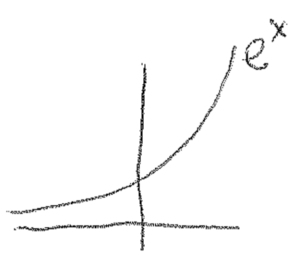
$$= 4 \cdot e^{-2x} + 4x \cdot (-2e^{-2x})$$

$$= 4e^{-2x} - 8xe^{-2x}$$

$$= 4e^{-2x}(1 - 2x)$$

Solve for x $\quad 4e^{-2x}(1-2x) = 0$

$4e^{-2x} = 0 \quad$ or $\quad 1 - 2x = 0$

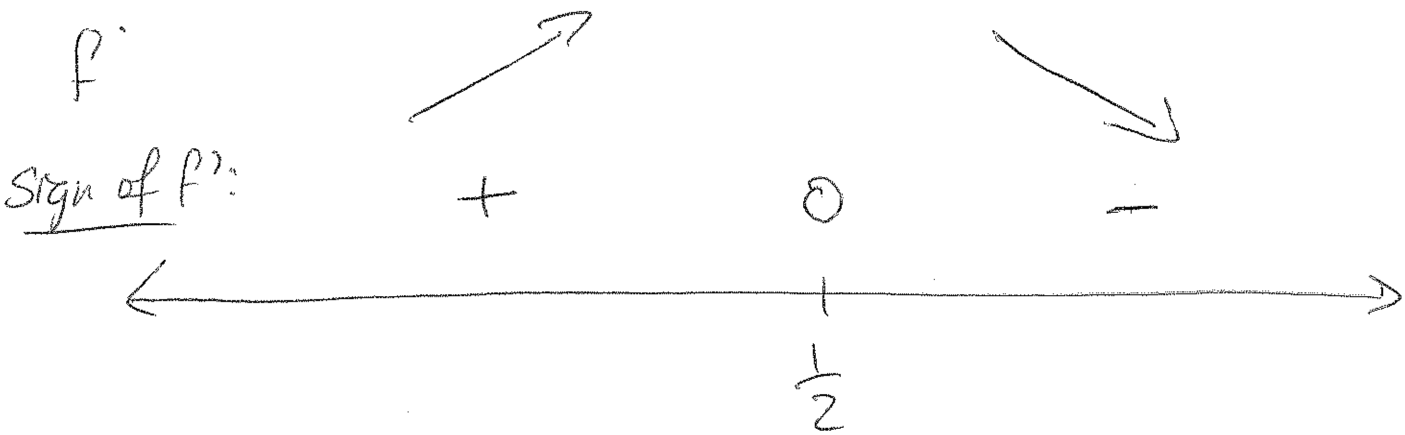



No soln

$$1 = 2x$$

$$x = \frac{1}{2}.$$

So $f$ has one critical point at $x = \frac{1}{2}$.




$$\boxed{\text{So } f \text{ has local max at } x = \tfrac{1}{2}.}$$