Announcements

- HW 7 due ~~tonight~~ tomorrow 11pm
- Test 2 Review: Monday Oct 18
- Test 2: Wed Oct 20 : 2.3-2.5, 3.1-3.4, 4.1, 4.

4.2: Concavity and Inflection Points

def A point at which the concavity of a function changes is an *inflection point*.

To find the inflection points of f:

1. Compute $f''(x)$
2. Make a sign chart for $f''(x)$.
3. Fill in concavity information
4. The inflection points are where the concavity changes.

Ex: Find the inflection points of $f(x) = x^4 - 2x^2 - 3x + 1$.

Soln:

- $f'(x) = \frac{d}{dx}\left[x^4 - 2x^2 - 3x + 1\right]$

① $= 4x^3 - 4x - 3$

- $f''(x) = \frac{d}{dx}\left[4x^3 - 4x - 3\right]$

$= 12x^2 - 4$

②: Sign chart for $f''(x)$;

and ③:

- Set $f''(x) = 0$:

$12x^2 - 4 = 0$

$12x^2 = 4$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{1}}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$\boxed{\text{Recall: } f''(x) = 12x^2 - 4}$

Sign chart $f''(x)$

Concavity	$\cup$		$\cap$		$\cup$
$f''(x)$	$+$	0	$-$	0	$+$
	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1

④: $\boxed{f(x) \text{ has inflection points at } -\frac{\sqrt{3}}{3} \text{ and } \frac{\sqrt{3}}{3}.}$

<u>Ex</u>: Find the inflection points of $f(x) = x^4 - 8x^3 + 24x^2$.

<u>Soln</u>: $f'(x) = \frac{d}{dx}\left[x^4 - 8x^3 + 24x^2\right]$

$$= 4x^3 - 24x^2 + 48x$$

$$f''(x) = \frac{d}{dx}\left[4x^3 - 24x^2 + 48x\right]$$

$$= 12x^2 - 48x + 48.$$

<u>Solve for x in:</u> $12x^2 - 48x + 48 = 0$

$$12(x^2 - 4x + 4) = 0$$

$$12(x-2)(x-2) = 0$$

$$\boxed{\begin{aligned} f''(x) &= 12(x-2)(x-2) \\ &= 12(x-2)^2 \end{aligned}}$$

$\cancel{12 = 0}$ $x - 2 = 0$ or $x - 2 = 0$

$x = 2$ $x = 2$.

Concavity		$\cup$		$\cup$
Sign chart for $f''(x)$	$f''(x)$	$+$	0	$+$
		0	2	3

$\boxed{\text{So } f \text{ has no inflection points.}}$

Ex: Let $f(x) = x^2 e^{-x}$.

(a) Find and classify the critical points of $f(x)$.

Soln $f'(x) = \frac{d}{dx}\left[x^2 e^{-x}\right]$

$$= \frac{d}{dx}\left[x^2\right] \cdot e^{-x} + x^2 \frac{d}{dx}\left[e^{-x}\right]$$

$$= 2x \cdot e^{-x} + x^2(-1)e^{-1x}$$

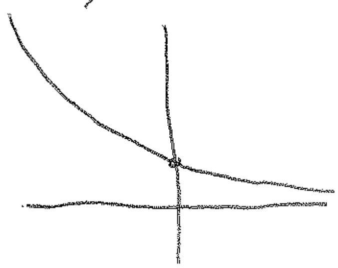
$$= 2xe^{-x} - x^2 e^{-x}$$

Set $f'(x)=0$: $2xe^{-x} - x^2e^{-x} = 0$

$$e^{-x}(2x - x^2) = 0$$

$$e^{-x}\cdot x(2 - x) = 0$$

$\cancel{e^{-x}=0}$ or $x=0$ or $2-x=0$

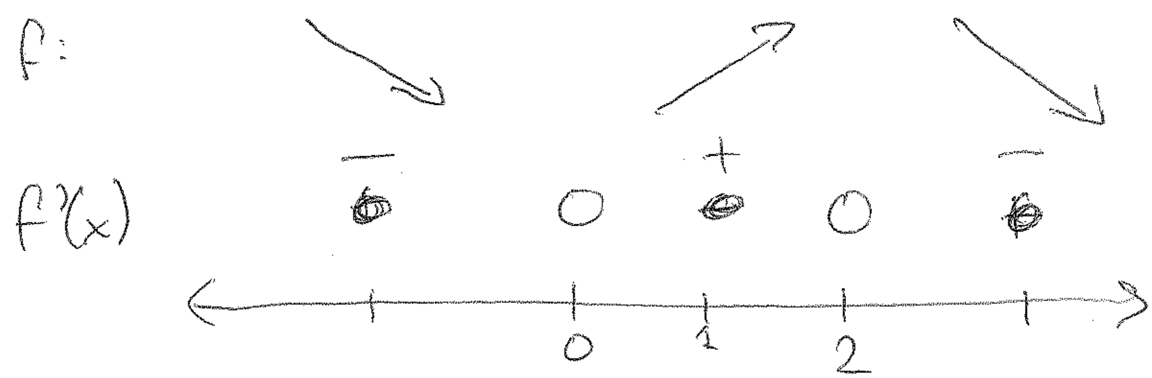


no soln

$x=0$ $\quad$ $x=2$

$$\boxed{\begin{aligned} f'(x) &= e^{-x}\cdot x \cdot \\ &\quad e^{-x}\cdot x\cdot(2-x) \end{aligned}}$$

- Critical points: 0 and 2.
- Use FDT: make a sign chart for $f'(x)$:



- So f has a local min at $x=0$
  local max at $x=2$.

(b) Find the inflection points of f(x).

(Recall $f(x) = x^2 e^{-x}$

$f'(x) = 2x e^{-x} - x^2 e^{-x}$

$= e^{-x} \cdot x \cdot (2-x)$.)

Soln: $f''(x) = \frac{d}{dx}\left[2xe^{-x} - x^2 e^{-x}\right]$

$= \frac{d}{dx}\left[2x e^{-x}\right] - \frac{d}{dx}\left[x^2 e^{-x}\right]$

$= 2 \cdot e^{-x} + 2x \frac{d}{dx}\left[e^{-x}\right] - \left(2xe^{-x} - x^2 e^{-x}\right)$

$= 2e^{-x} + 2x(-1)e^{-x} - 2xe^{-x} + x^2 e^{-x}$

$= 2e^{-x} - 2xe^{-x} - 2xe^{-x} + x^2 e^{-x}$

$= e^{-x}(2 - 2x - 2x + x^2)$

$= e^{-x}(x^2 - 4x + 2)$

(2). Solve for x in $e^{-x}(x^2 - 4x + 2) = 0$

$e^{-x} = 0$ or $x^2 - 4x + 2 = 0$

No Soln   Use quad. formula: