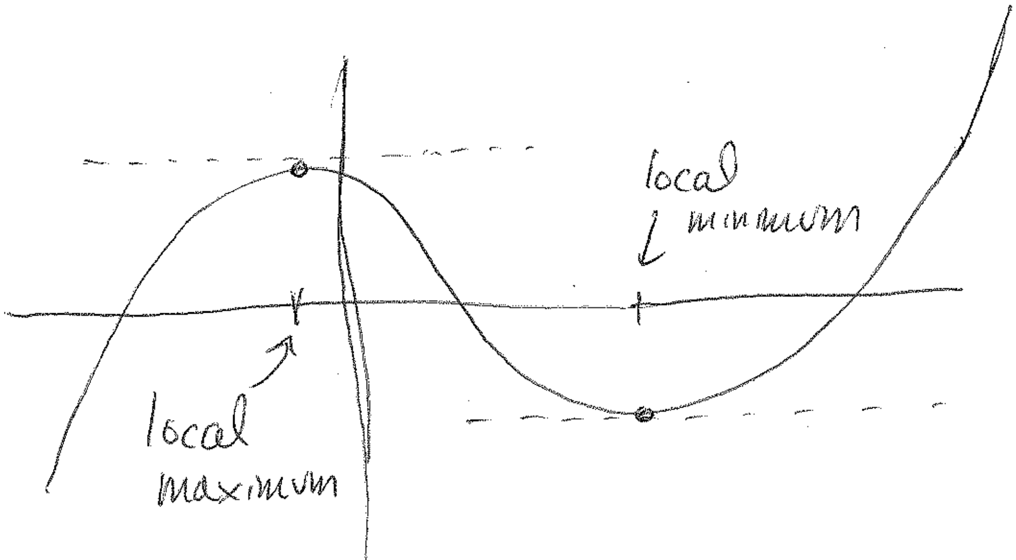
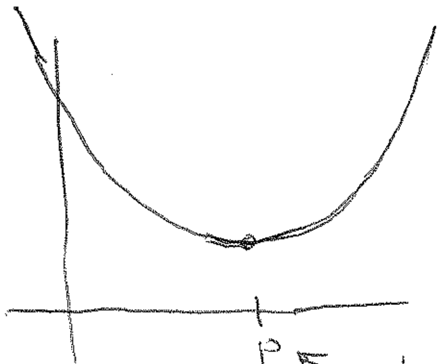
Today: Section 4.1: Local minima / maxima



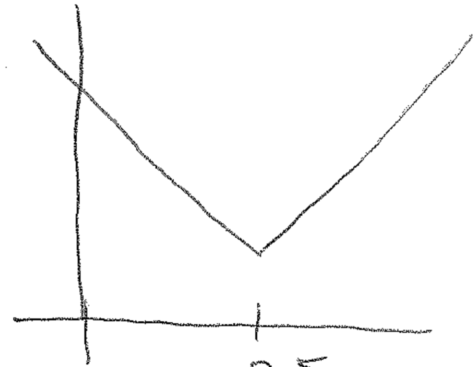
- ~~If~~ A function f has a local minimum at p if $f(p)$ is less than the value of f at points near p.
- A function f has a local maximum at p if $f(p)$ is greater than the value of f at points near p.

def: A point p is a critical point of f if $f'(p) = 0$ or $f'(p)$ is undefined.



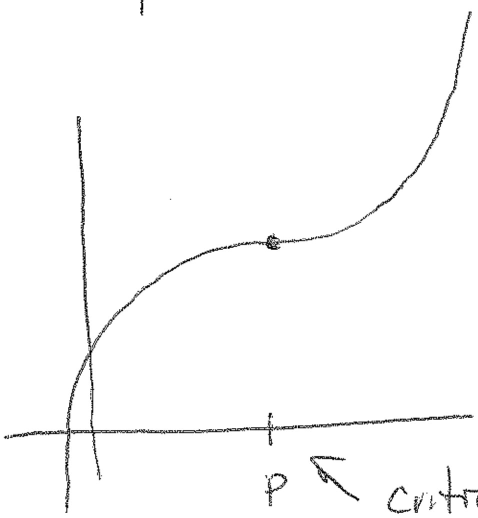
P ← critical point

$f'(p) = 0$



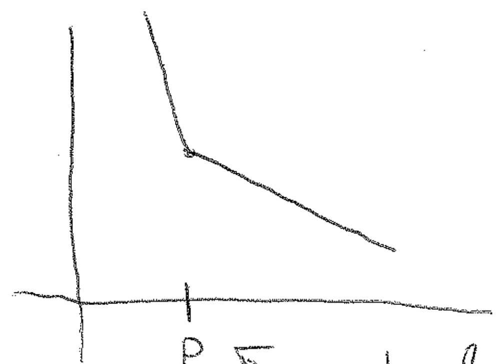
P ← critical point

$f'(p)$ is undefined



P ← critical point

$f'(p) = 0$



P ← critical point

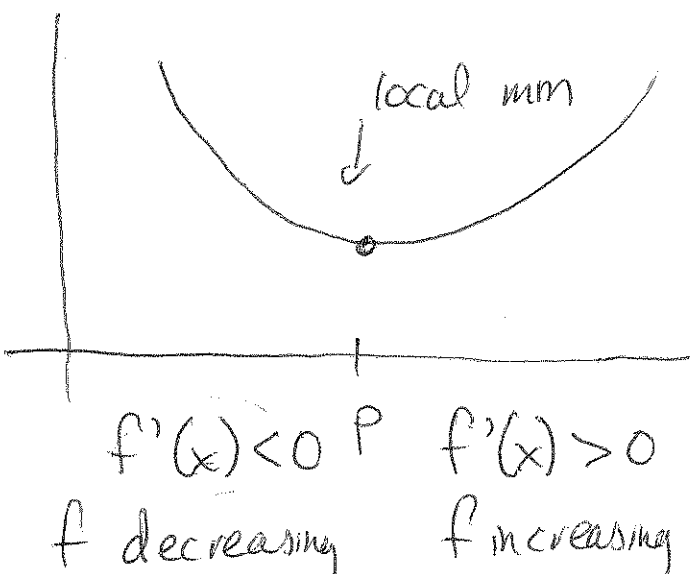
$f'(p)$ is undefined.

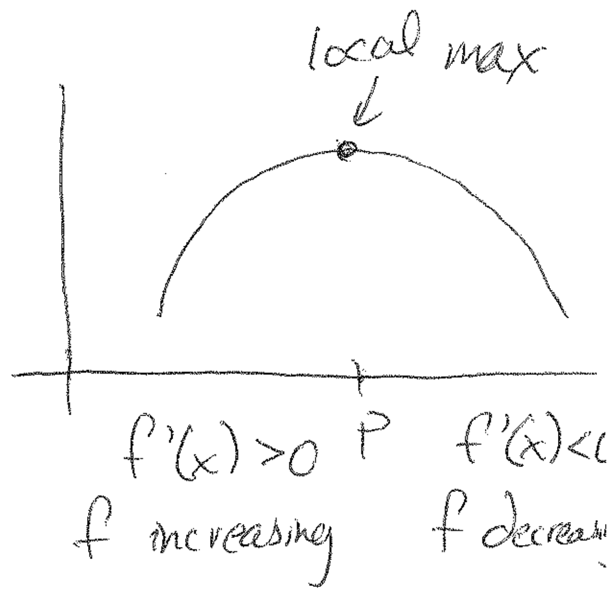
Fact: If $f(x)$ is continuous on $[a, b]$, then every local minima and local maxima occurs either at a critical point or at the endpoints (a and/or b)

First Derivative Test: Let p be a critical pt. of f.

- If $f'(x)$ changes signs from negative to positive as x passes through p, then p is a local minimum.

- If $f'(x)$ changes signs from positive to negative as x passes through p, then p is a local maximum.

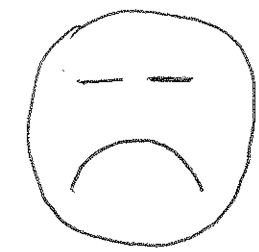




- If $f'(x)$ does not change signs as x passes through p, then p is neither a local min. nor a local max.

Second Derivative Test: Let p be a critical point of f.

- If $f''(p) > 0$, then p is a local min



- If $f''(p) < 0$, then p is a local max.

- If $f''(p) = 0$, then the S.D.T. is inconclusive; must use the F.D.T.

Ex: Find and classify the critical points of
$f(x) = x^3 + 2x^2 + x - 5$.

Soln **Step 1:** Find the critical points.

- $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[x^3 + 2x^2 + x - 5]$

$$= 3x^2 + 4x + 1$$

- Solve for x in $3x^2 + 4x + 1 = 0$

$$(3x + 1)(x + 1) = 0$$

- $3x + 1 = 0$ or $x + 1 = 0$

$3x = -1 \qquad\qquad x = -1.$

$x = \frac{-1}{3}$

- Critical points are $-\frac{1}{3}$ and -1.

<u>Step 2</u>: Classify the critical points.

- $f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}\left[3x^2 + 4x + 1\right]$

$= 6x + 4$

<u>$-\frac{1}{3}$</u>: $f''\left(-\frac{1}{3}\right) = 6\left(-\frac{1}{3}\right) + 4$

$= -2 + 4 = 2 > 0$

$\Longrightarrow$ $\boxed{-\frac{1}{3} \text{ is a local minimum.}}$

<u>-1</u>: $f''(-1) = 6(-1) + 4 = -6 + 4 = -2 < 0$

$\Longrightarrow$ $\boxed{-1 \text{ is a local maximum}}$

Ex Find and classify the critical points of $f(x) = x^3 - 3x^2 + 3x + 2$.

Soln: Step 1: Find critical points:

- $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}\left[x^3 - 3x^2 + 3x + 2\right]$

$$= 3x^2 - 6x + 3$$

- Solve for x in $3x^2 - 6x + 3 = 0$

$$3(x^2 - 2x + 1) = 0$$

$$3(x - 1)(x - 1) = 0$$

$x - 1 = 0$ or $x - 1 = 0$

$x = 1$ $\quad$ $x = 1$

- So there is one critical point of f at $x = 1$.

Step 2: Classify the critical points

- $f''(x) = \frac{d}{dx}[f'(x)] = \frac{d}{dx}\left[3x^2 - 6x + 3\right]$

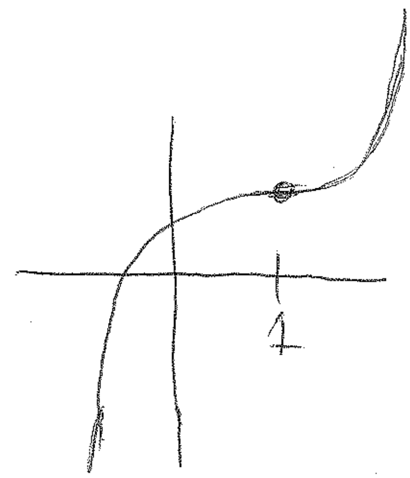
$$= 6x - 6$$

$\underline{1}$: $f''(1) = 6(1) - 6 = 0$

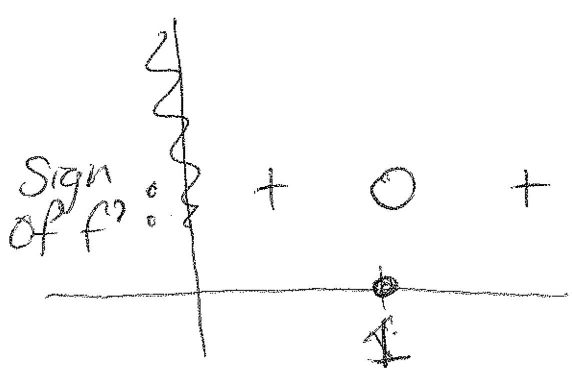
 $\Longrightarrow$ S.D.T. is inconclusive.
Need F.D.T.

To apply the F.D.T, need to know the sign of $f'(x)$ around the critical pt 1.

$$f'(x) = 3x^2 - 6x + 3$$
$$= 3(x-1)(x-1)$$
$$= 3\underbrace{(x-1)^2}_{\text{square ; always } \geq 0.}$$



$\Longrightarrow$ $f'(x) \geq 0$ for all x.



Sign of $f'(x)$ does not change as x passes through 1,
so F.D.T $\Longrightarrow$ $x=1$ is neither a local min nor a local max