Announcements

- DROP DEADLINE TOMORROW!
- HW6 due Tomorrow
- Quiz 6 out Tomorrow; due Monday Oct 11
- OH Friday: 11am – noon.

Ex: $\frac{d}{dt}\left[\left(\underbrace{4t^2 + 8\ln(t)}_{z}\right)^3\right]$

$$= \frac{d}{dz}\left[z^3\right] \cdot \frac{d}{dt}\left[4t^2 + 8\ln(t)\right]$$

$$= 3z^2 \cdot \left(\frac{d}{dt}\left[4t^2\right] + \frac{d}{dt}\left[8\ln t\right]\right)$$

$$= 3z^2 \cdot \left(8t + \frac{8}{t}\right)$$

$$= \boxed{3\left(4t^2 + 8\ln(t)\right)^2 \cdot \left(8t + \frac{8}{t}\right)}$$

Ex: $\frac{d}{dx}\left[\ln\left(\underbrace{2+e^{(x^2)}}_{z}\right)\right]$

chain rule

$$= \frac{d}{dz}\left[\ln(z)\right] \cdot \frac{d}{dx}\left[2+e^{(x^2)}\right]$$

$$= \frac{1}{2+e^{(x^2)}} \cdot \left(\frac{d}{dx}[2] + \frac{d}{dx}\left[e^{\overbrace{(x^2)}^{z}}\right]\right)$$

chain Rule

$$= \frac{1}{2+e^{(x^2)}} \cdot \left(0 + e^{(x^2)} \cdot \frac{d}{dx}\left[x^2\right]\right)$$

$$= \frac{1}{2+e^{(x^2)}} \cdot \left(e^{(x^2)} \cdot 2x\right)$$

$$= \boxed{\frac{2x\, e^{(x^2)}}{2+e^{(x^2)}}}$$

Section 3.4: Product Rule and Quotient Rule

$$\boxed{\cdot\ \frac{d}{dx}\left[f(x)\cdot g(x)\right] = f'(x)\cdot g(x) + f(x)\cdot g'(x)}$$

$\Longrightarrow$ CAUTION: Don't try $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g'(x)$

ILLEGAL!

- Recall $\frac{d}{dx}[x^2] = 2x$

ILLEGAL!

- $\frac{d}{dx}[x \cdot x] = \frac{d}{dx}[x] \cdot \frac{d}{dx}[x]$

$= 1 \cdot 1 = 1$

Correct

- $\frac{d}{dx}[x \cdot x] = \frac{d}{dx}[x] \cdot x + x \cdot \frac{d}{dx}[x]$

$= 1 \cdot x + x \cdot 1 = 2x$

Ex: $\frac{d}{dx}[\sqrt{x} \cdot e^x] = \frac{d}{dx}[\sqrt{x}] \cdot e^x + \sqrt{x} \cdot \frac{d}{dx}[e^x]$

$= \frac{d}{dx}[x^{1/2}] \cdot e^x + \sqrt{x} \cdot e^x$

$= \boxed{\frac{1}{2} x^{-1/2} \cdot e^x + \sqrt{x} \cdot e^x}$

$= \frac{1}{2} \cdot \frac{1}{x^{1/2}} \cdot e^x + \sqrt{x} \cdot e^x$

$= \boxed{\frac{e^x}{2\sqrt{x}} + \sqrt{x}\, e^x}$

Ex: $\frac{d}{dt}\left[t^{8.2}\ln(2t+1)\right]$

$$= \frac{d}{dt}\left[t^{8.2}\right]\cdot \ln(2t+1) + t^{8.2}\cdot\frac{d}{dt}\left[\ln(\overbrace{2t+1}^{z})\right]$$

$$= 8.2\,t^{7.2}\cdot\ln(2t+1) + t^{8.2}\cdot\frac{1}{2t+1}\cdot\frac{d}{dt}\left[2t+1\right]$$

$$= \boxed{8.2\,t^{7.2}\cdot\ln(2t+1) + t^{8.2}\cdot\frac{1}{2t+1}\cdot 2}$$

- Quotient Rule:

$$\boxed{\begin{aligned}\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] &= \frac{\text{low}\cdot d\text{high} - \text{high}\cdot d\text{low}}{(\text{low})^2}\\ &= \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}\end{aligned}}$$

- Ex $\frac{d}{dx}\left[\frac{3x^2}{4x^3+1}\right] = \frac{(4x^3+1)\frac{d}{dx}\left[3x^2\right] - 3x^2\frac{d}{dx}\left[4x^3+1\right]}{(4x^3+1)^2}$

$$= \frac{(4x^3+1)\cdot 6x - 3x^2\cdot 12x^2}{(4x^3+1)^2}$$

$$= \boxed{\frac{(4x^3+1)\cdot 6x - 36x^4}{(4x^3+1)^2}}$$

- $\underline{Ex}$ $$\frac{d}{dx}\left[\frac{1}{\ln(x+5)}\right] = \frac{\ln(x+5)\cdot 0 - 1\cdot\frac{d}{dx}[\ln(\overbrace{x+5}^{z})]}{(\ln(x+5))^2}$$

$$= \frac{0 - \frac{1}{x+5}\cdot\frac{d}{dx}[x+5]}{(\ln(x+5))^2}$$

$$= \frac{-\frac{1}{x+5}\cdot 1}{(\ln(x+5))^2} = -\frac{1}{x+5}\cdot\frac{1}{(\ln(x+5))^2}$$

$$= \boxed{-\frac{1}{(x+5)(\ln(x+5))^2}}$$

- $\underline{Ex}$ $$\frac{d}{dx}\left[\frac{1}{\ln(x+5)}\right] = \frac{d}{dx}\left[(\underbrace{\ln(x+5)}_{z})^{-1}\right]$$

$$= \frac{d}{dz}[z^{-1}]\cdot\frac{d}{dx}\left[\ln(\underbrace{x+5})\right]$$

$$= -1\cdot z^{-2}\cdot\frac{1}{x+5}\cdot\frac{d}{dx}[x+5]$$

$$= -1\cdot(\ln(x+5))^{-2}\cdot\frac{1}{x+5}\cdot 1 = -1\cdot\frac{1}{(\ln(x+5))^2}\cdot\frac{1}{x+5}$$