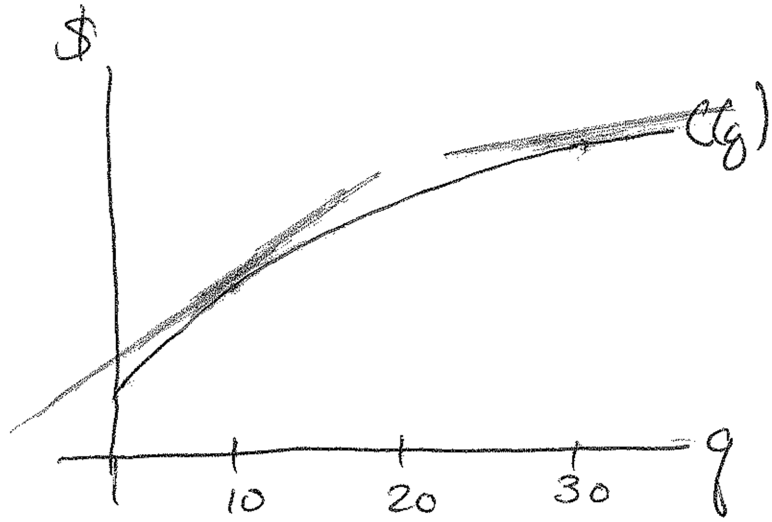
Section 2.5: Marginal Cost and Marginal Revenue

- $C(q)$ - cost of producing q units of some good
- $R(q)$ - revenue received when q units are produced

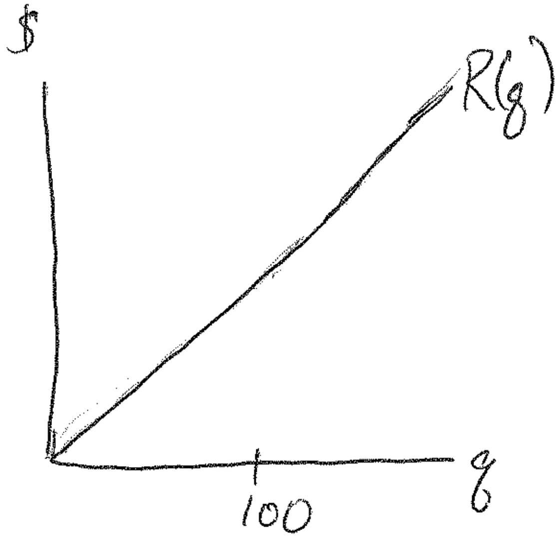
<u>def</u> The marginal cost (or MC) ~~of producing~~ ~~the~~ at production level q is $C'(q)$.

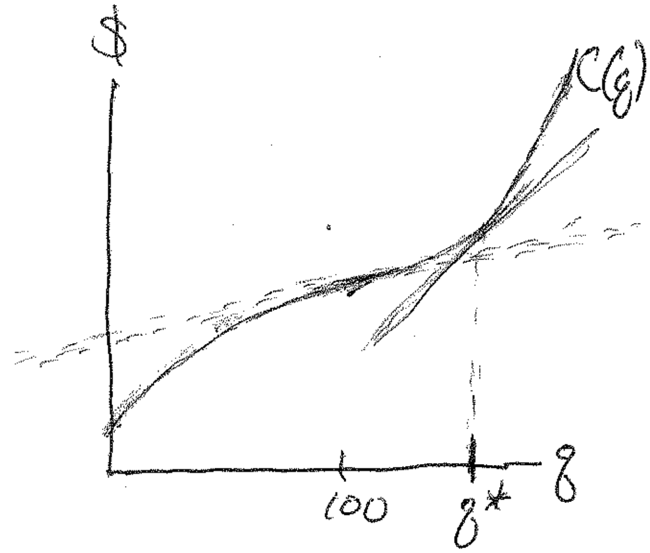


<u>def</u> The marginal revenue (or MR) at production level q is $R'(q)$.

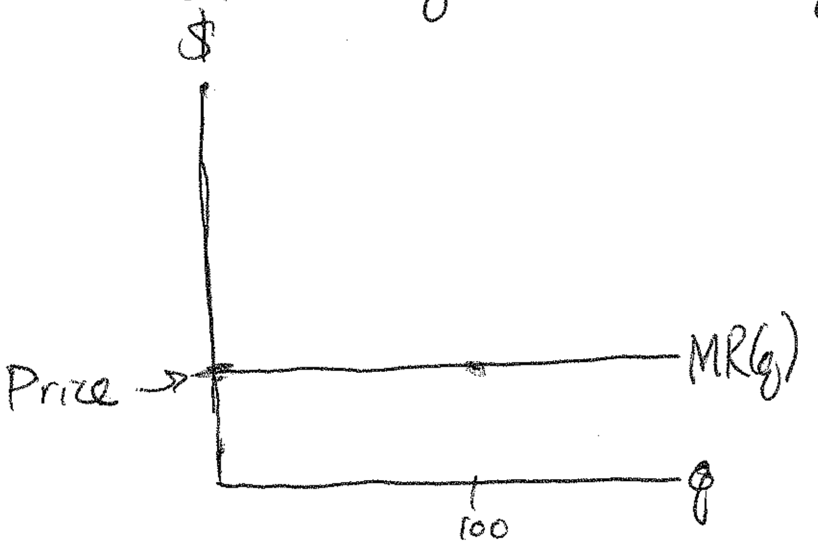
<u>Q</u>: If my production level is q, should I increase production? Yes if $MR(q) > MC(q)$.
No otherwise.

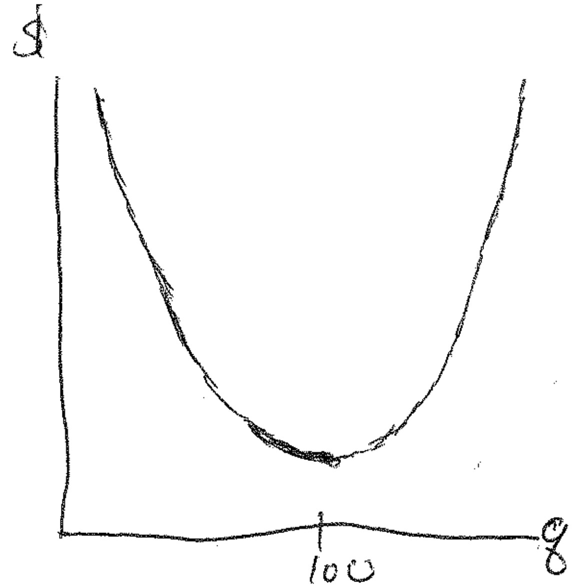
Ex 4





- Sketch MR(q) and MC(q).





- Suppose my production level is $q = 100$. Should I increase production? Yes, since $MR(100) > MC(100)$.

- At q^*, $MR(q^*) = MC(q^*)$, so I should stop my production at q^* units.

- $\left(MR(q) = R'(q) \;;\; MC(q) = C'(q) \right)$

Announcements

- HW 5 (2.3-2.5) due Fri
- OH Friday moved to 4pm-5pm
- Drop deadline: Oct 7.

Section 2.5:

Ex (#11) $C(q)$ is a cost function.

(a) If $C(50) = 4300$ and $MC(50) = 24$.

Estimate $C(52)$.

Linear approximation / Tangent line: of a fn f at $x = a$

$$\boxed{y = f(a) + f'(a)(x-a)}$$

$$y = f(a) + f'(a)\Delta x$$

Point/Slope of a line through $(a, f(a))$ with slope $f'(a)$.

Soln (a) Use tangent line eqn ~~at~~ for $a = 50$, $f = C$.

$$y = f(a) + f'(a)(x-a)$$

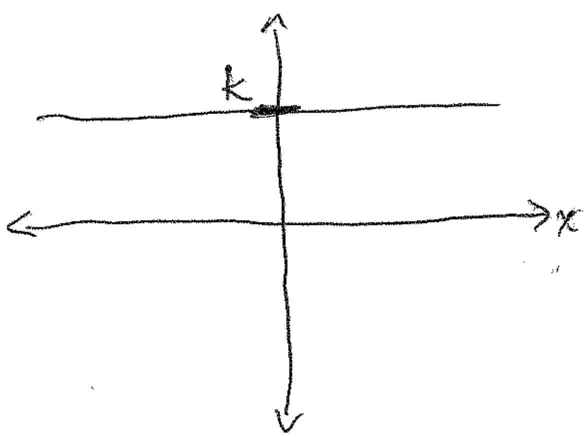
$$\Rightarrow \quad y = C(50) + C'(50)(x-50)$$

$$y = 4300 + 24(x-50)$$

$$C(52) \approx 4300 + 24(52-50)$$
$$\approx 4300 + 24 \cdot 2$$
$$\approx 4300 + 48 = \boxed{4348}$$

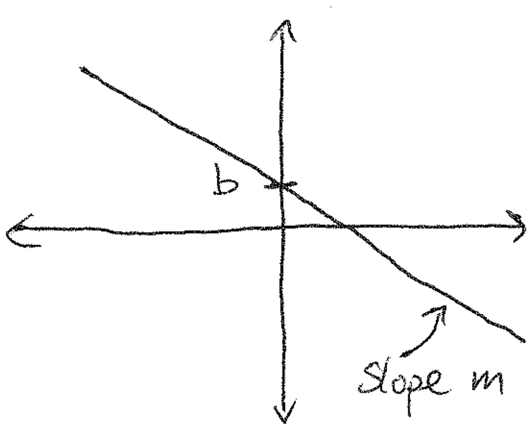
Section 3.1: Shortcuts for derivatives

- Constant function: Let k be a constant.



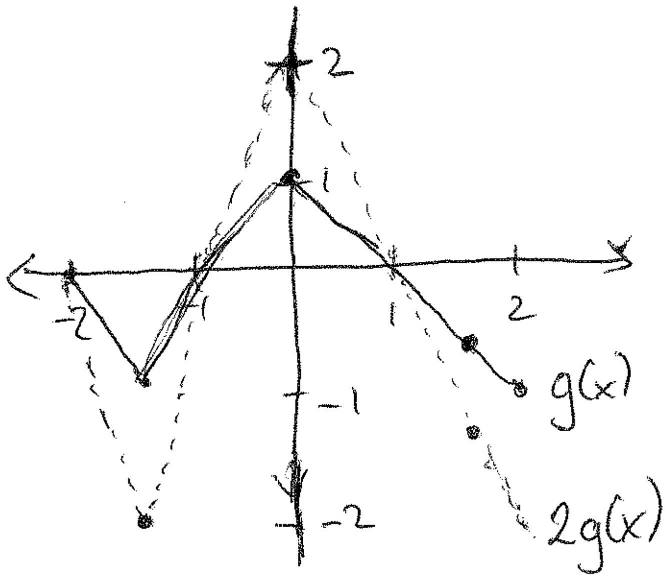
If $f(x) = k$, then $f'(x) = 0$ || $\frac{d}{dx}[k] = 0$

- Linear functions:



If $f(x) = mx + b$, then $f'(x) = m$ || $\frac{d}{dx}[mx+b] = m$

- Constant multiples of functions:



If $f(x) = cg(x)$,

then $f'(x) = c\,g'(x)$.

$$\frac{d}{dx}[cg(x)] = c\frac{d}{dx}[g(x)]$$

- Sums of functions:

- If $f(x) = g(x) + h(x)$,

then $f'(x) = g'(x) + h'(x)$.

$$\frac{d}{dx}[g(x)+h(x)] = \frac{d}{dx}[g(x)] + \frac{d}{dx}[h(x)]$$

Ex: $$\frac{d}{dx}[27x - e^2] = \frac{d}{dx}[27x + (-e^2)]$$
$$= \frac{d}{dx}[27x] + \frac{d}{dx}[-e^2]$$
$$= 27 + 0$$
$$= \boxed{27}$$

Power Rule: Let n be a real number.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

Ex: $\frac{d}{dx}\left[4x^2\right] \underset{\text{multiple}}{\overset{\text{const}}{=}} 4\frac{d}{dx}\left[x^2\right]$

$\overset{\text{Power Rule}}{=} 4\left(2\cdot x^{2-1}\right)$

$= 4\left(2x^1\right) = 4(2x) = \boxed{8x}$

Ex $\frac{d}{dx}\left[2x^{-3.2}\right] = 2\frac{d}{dx}\left[x^{-3.2}\right]$

$= 2\left(-3.2\,x^{-3.2-1}\right)$

$= 2\left(-3.2\,x^{-4.2}\right)$

$= \boxed{-6.4x^{-4.2}}$

Ex $\frac{d}{dx}\left[-\frac{1}{x}\right] = \frac{d}{dx}\left[-x^{-1}\right]$

$= (-1)\cdot\frac{d}{dx}\left[x^{-1}\right]$

$= (-1)\cdot\left(-1\,x^{-1-1}\right)$

$= (-1)\left(-1\,x^{-2}\right) = x^{-2} = \boxed{\frac{1}{x^2}}$

Sum Rule

Ex $\frac{d}{dx}\left[x + 2\sqrt{x}\right] = \frac{d}{dx}[x] + \frac{d}{dx}\left[2\sqrt{x}\right]$

$= 1 + \frac{d}{dx}\left[2x^{1/2}\right]$

$= 1 + 2\frac{d}{dx}\left[x^{\frac{1}{2}}\right]$

$= 1 + 2\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)$

$= 1 + 2\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$= 1 + x^{-\frac{1}{2}}$

$= 1 + \frac{1}{x^{1/2}}$

$= \boxed{1 + \frac{1}{\sqrt{x}}}$