Announcements

- HW #5 out Tuesday due Friday

Today: 2.4: The second derivative

- <u>Operator</u> <u>Notation</u> $\frac{d}{dx}\left[\underbrace{x^2 + 3x}\right]$

$\underbrace{\qquad}$ Take the derivative of the fn $f(x) = x^2 + 3x$

<u>Ex:</u>

$f(x) = x^2 + 3x$

$f'(x) = 2x + 3$

$y = x^2 + 3x$

$\frac{d}{dx}[y] = \frac{d}{dx}[x^2 + 3x]$

$= 2x + 3$

- We write $\frac{dy}{dx}$ for $\frac{d}{dx}[y]$.

Second Derivatives

- The derivative of a function $f(x)$ is a function $f'(x)$.

- We can take the derivative of $f'(x)$.

- This function is called the <u>second</u> <u>derivative</u>, and its denoted by $f''(x)$ or $\frac{d^2y}{dx^2}$.

= What does $f''(x)$ tell us about $f'(x)$ and $f(x)$?

- $f''(x) > 0$ on $[a,b]$ $\Longleftrightarrow$ $f'(x)$ is increasing on $[a,b]$

  $\Longleftrightarrow$ the slope of f is increasing on $[a,b]$.

  $\Longleftrightarrow$ f is "bending up"

  $\Longleftrightarrow$ f is concave up on $[a,b]$

- $f''(x) < 0$ on $[a,b]$ $\iff$ $f'(x)$ is decreasing on $[a,b]$

$\iff$ Slope of f is decreasing

$\iff$ f is bending down

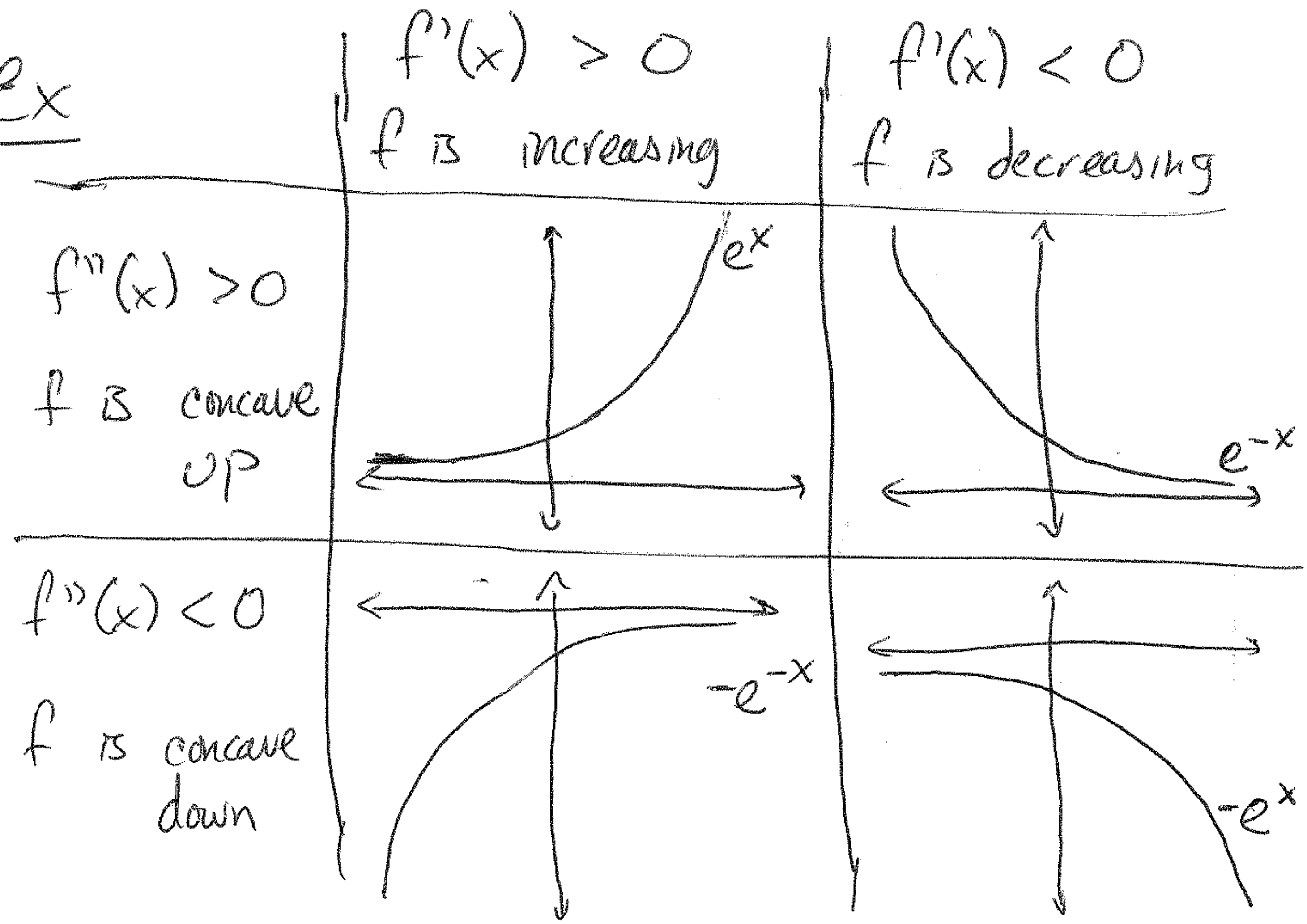
$\iff$ f is concave down on $[a,b]$

<u>Note:</u> • The sign of $f'(x)$ tells us if f is increasing/decreasing

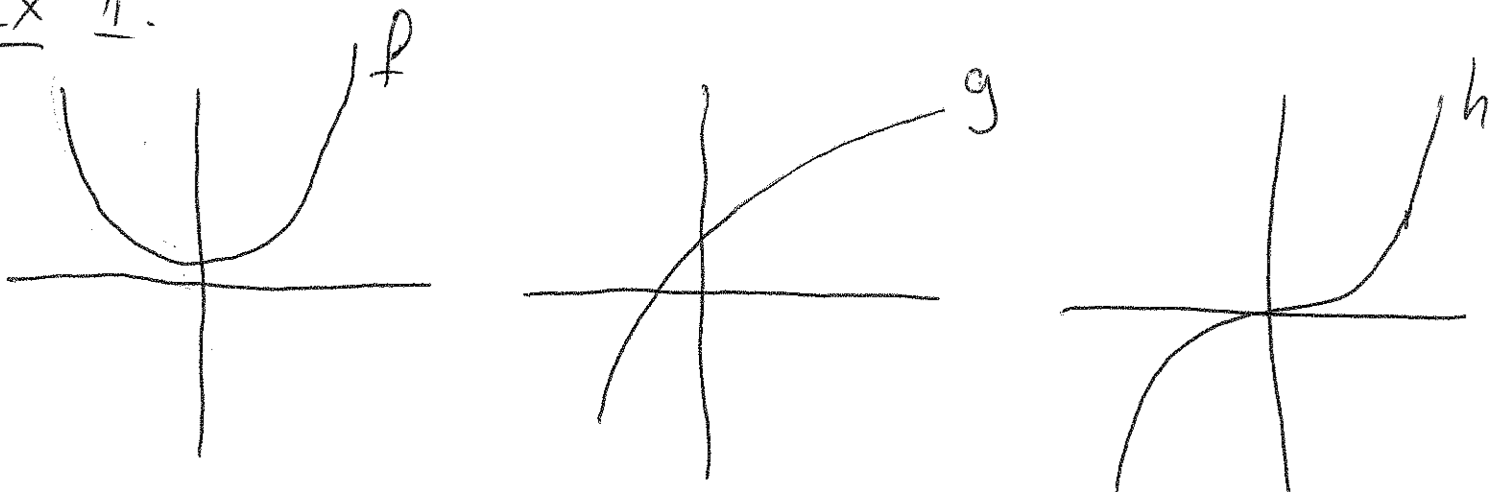
• The sign of $f''(x)$ tells us about the concavity of f.

$\Longrightarrow$ These properties of f are independent in that all 4 combinations are possible.

Ex



Ex 1:

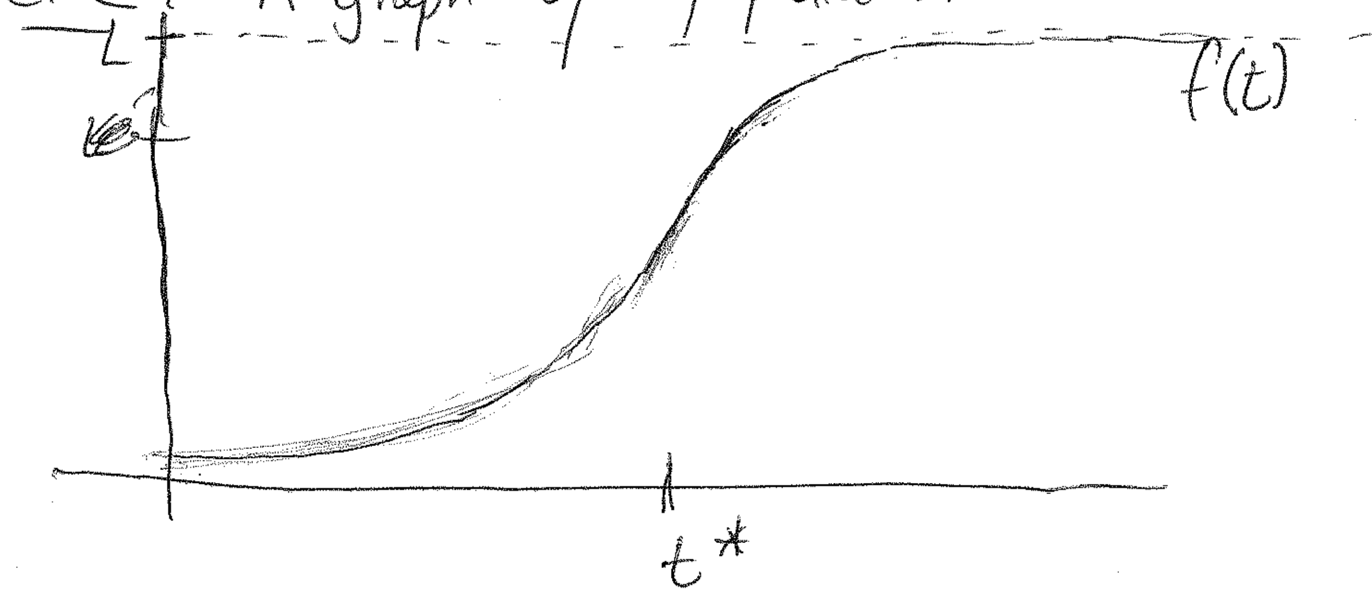


- Describe the sign of the second derivatives.

Soln

- $f''(x) > 0$
- $g''(x) < 0$
- $h''(x) < 0$ on $(-\infty, 0)$
- $h''(x) > 0$ on $(0, \infty)$

Ex 2: A graph of Population



- Interpret/explain the significance of t^* in terms of the model, $f(t)$, $f'(t)$, and $f''(t)$.

Soln:

- $f(t)$ has a change in concavity at t^* from concave up to concave down.

- Slope of f increases on $(0, t^*)$ and decreases on (t^*, ∞)

- $f'(t)$ has its maximum at t^*

- The rate of population growth is ~~fastest~~ greatest at t^*.