Announcements

see 011

- ~~Hw~~ Quiz 4 due tonight 11 pm
- Test #1 Wednesday 1.1–1.9, 2.1, 2.2

- Continuous Rate Prob, half-life
- Sec 1.4
- Graphing derivatives - finding derivatives using "slope of tan."
- Finding derivs using avg rate of change.

Ex: 78 kg of radioactive material is released into the environment. After 10 hours, 70 kg of radioactive material remains. What is the half-life of the material?

Soln:

- $\boxed{P(t) = P_0 e^{kt}}$ use this formula for continuous, exp rates of change.

Recall: discrete rate

$P(t) = P_0 (1+r)^t$

- $P(t) = 78 e^{kt}$
- Find k by solving for k in:

$70 = 78 e^{k(10)}$

$$\frac{70}{78} = e^{10k}$$

$$\ln\left(\frac{70}{78}\right) = 10k$$

$$k = \frac{\ln\left(\frac{70}{78}\right)}{10} = -0.011$$

- $P(t) = 78e^{(-0.011)t}$

- Solve for t in the eqn:

$$39 = 78e^{(-0.011)t}$$

$$\frac{1}{2} = e^{(-0.011)t}$$

$$\ln\left(\frac{1}{2}\right) = -0.011t$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.011} = \boxed{63.01 \text{ hours}}$$

~~#1~~ Sec 1.4 # 15

- Fixed overhead: \$650,000
- Variable costs: \$20 per shoe
- Sale price: \$70 (each shoe)

(a) Find total cost $C(q)$

total revenue $R(q)$, and

total profit $P(q)$

Soln:

- $C(q) = 20q + 650{,}000$
- $R(q) = 70q$
- $P(q) = R(q) - C(q)$

$$= 70q - (20q + 650{,}000)$$

$$= 50q - 650{,}000$$

(b) Find marginal cost, marginal revenue, and marginal profit.

Soln:

- Marginal cost: \$20
- Marginal Revenue: \$70
- Marginal Profit: \$50

(c) How many shoes must be sold to profit?

$$0 = 50q - 650{,}000$$

$$650{,}000 = 50q$$

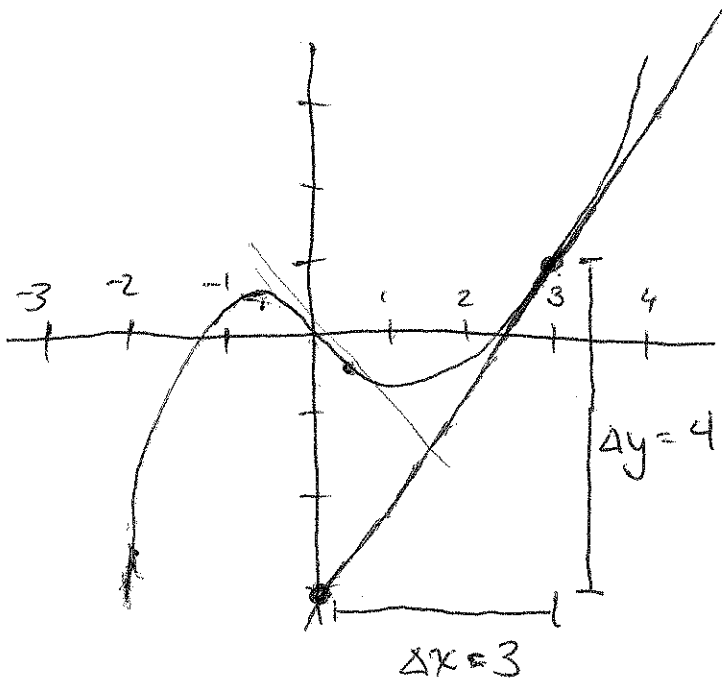
$$q = \frac{650 \cdot 1000}{50} = 13 \cdot 1000 = 13{,}000$$

break even

To profit: $\boxed{\text{13,001 shoes must be sold.}}$

Finding Derivatives

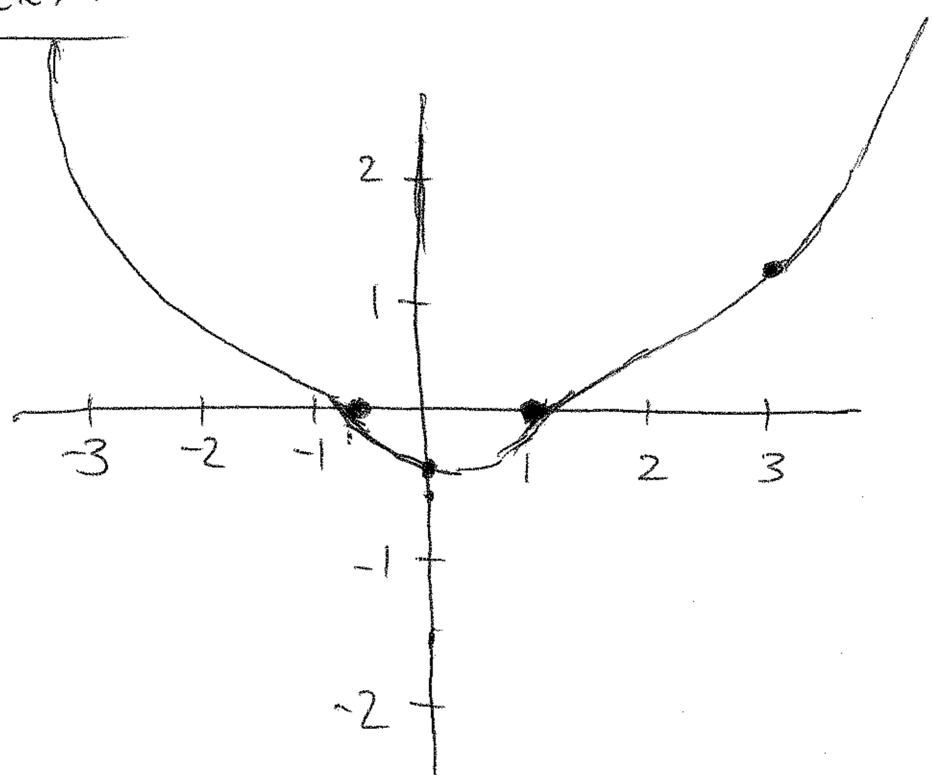
$f'(x)$ = slope of the tangent line at x



- Estimate the derivative of f at 3.
- $f'(3)$ = slope of the tangent line $= \frac{\Delta y}{\Delta x} = \frac{4}{3}$

- Estimate $f'(1)$
- $f'(1) = 0$

- Plot $f'(x)$:



Sec 013

- Present / Future
- ✓ Balance / interest (continuous vs. discrete)
- ✓ Estimate avg rate of change
- ✓ Graphing tangent lines / ~~g~~ sketching derivative of a graph
- ✓ f increasing / decreasing $\iff$ f' pos / neg
- <u>Half-life</u>:

- You have \$150 to deposit at a bank. One bank offers you 5% annual interest rate, compounded annually. Another bank offers 5% interest, compounded continuously. How much more would be in your account after 2 years, if you deposit with the continuously compounding bank?

<u>Soln</u>: → Continuously compounding bank:

$$P(t) = P_0 e^{kt}$$

$= 150\, e^{0.05t}$

- After 2 years, we have $P(2)$ dollars
- $P(2) = 150\, e^{(0.05)(2)} = 150(1.10517)$
  $= 165.78$

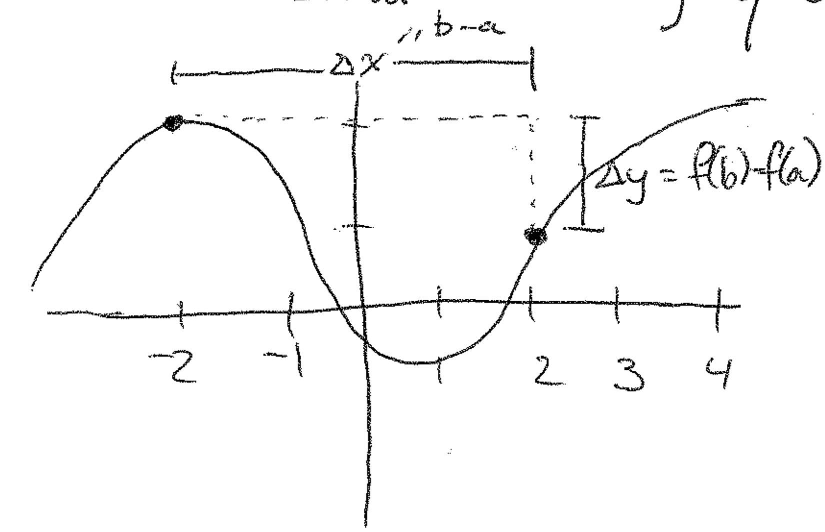
→ Interest compounded annually:

$$P(t) = P_0 (1+r)^t$$
$$= 150(1+0.05)^t$$
$$= 150(1.05)^t$$

- $P(2) = 150(1.05)^2 = 165.38$

- Difference: $\boxed{\text{\$0.40 more with continuously compounded interest}}$

Estimate the avg of change of f over $[-2, 2]$.

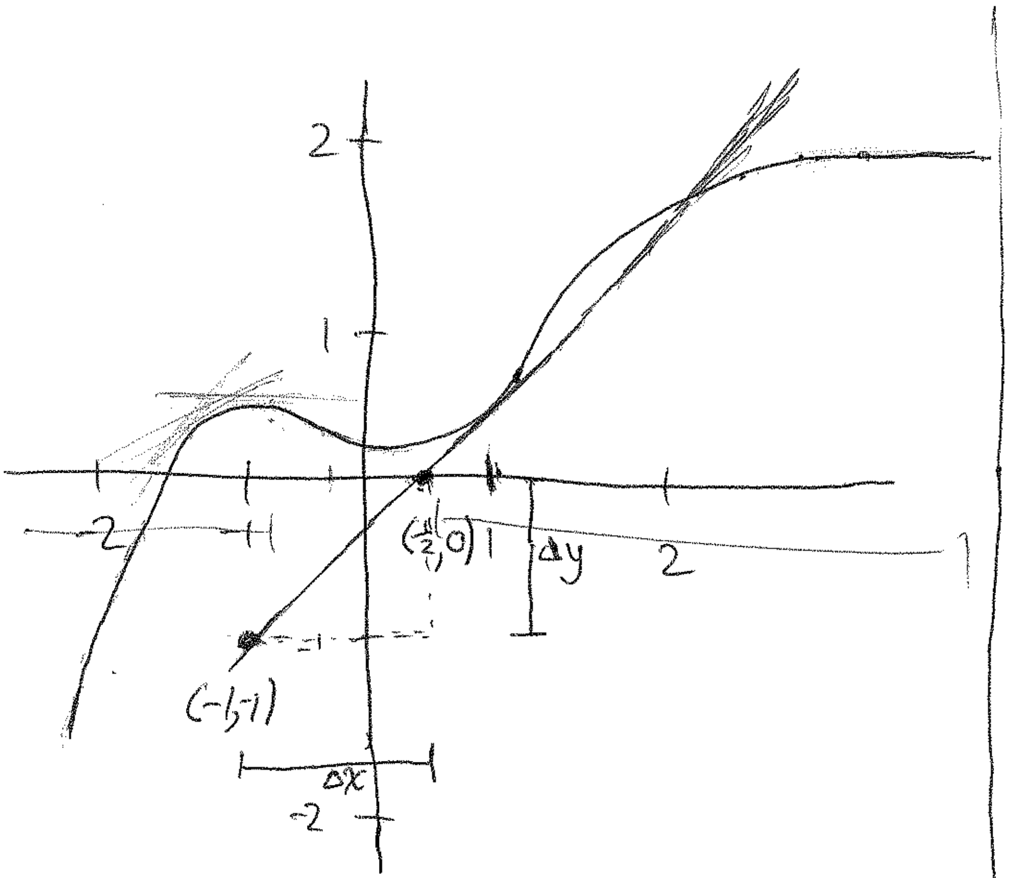


Soln:

$$\frac{f(b)-f(a)}{b-a} = \frac{f(2)-f(-2)}{2-(-2)}$$
$$= \frac{1-2}{4}$$
$$= \boxed{-\frac{1}{4}}$$

Sketching the derivative

- $f'(x) =$ slope of the tangent line at x

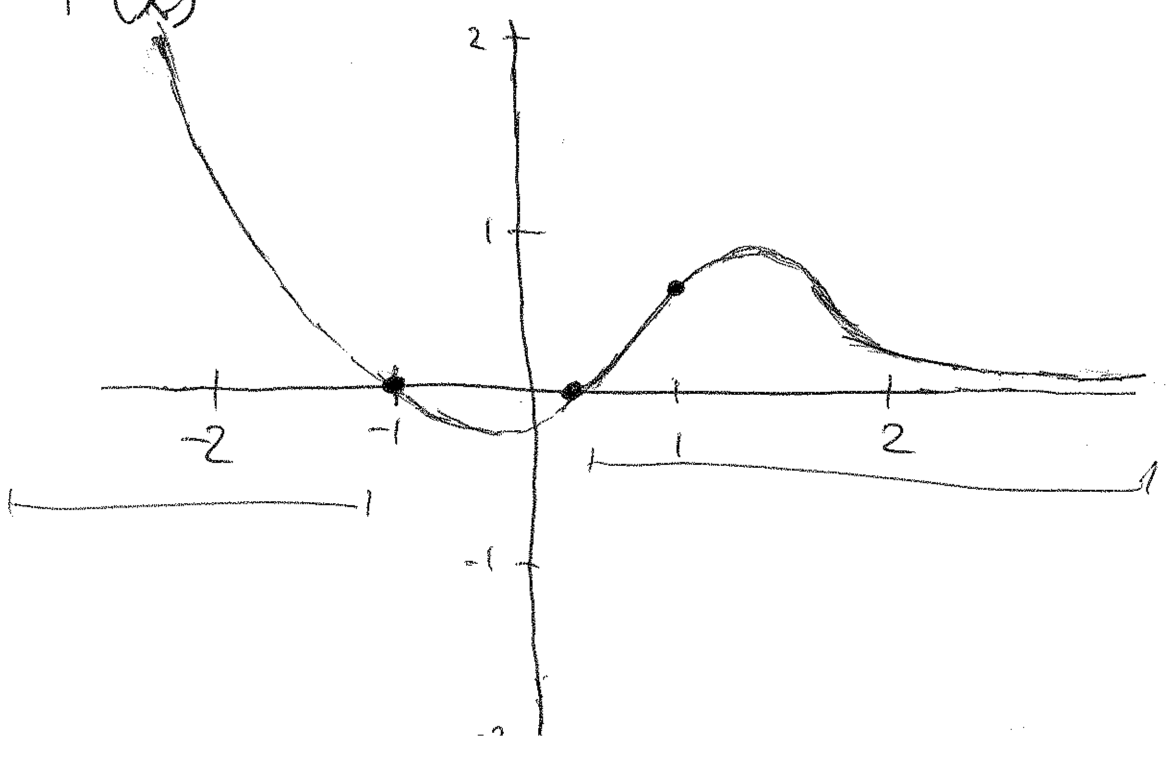


- Estimate $f'(1)$.

$$f'(1) = \frac{\Delta y}{\Delta x} = \frac{1}{1.5}$$

$$= \frac{10}{15} = \boxed{\frac{2}{3}}$$

- Sketch $f'(x)$



~~Half~~ Half-Life

- Suppose a radioactive material has a half-life of ~~@~~ 4 months. There's an accident at a research lab that causes 2 kg to be released. Before the scientists can return to work, at most $\frac{1}{50}$ kg of radioactive material can be present. How long will it take ~~if~~ if the material is allowed to decay naturally?

Soln:

$$P(t) = P_0 e^{kt}$$
$$P(t) = 2e^{kt}$$

- Need to find k:

$$\tfrac{1}{2}P_0 = 2e^{k(4)}$$
$$\tfrac{1}{2}(2) = 2e^{k4}$$
$$1 = 2e^{4\cdot k}$$
$$\frac{1}{2} = e^{4k}$$
$$\ln\left(\tfrac{1}{2}\right) = 4k$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{4}$$
$$= -0.1733$$

- $P(t) = 2e^{(-0.1733)t}$

- Solve for t in:

$$\frac{1}{50} = 2e^{(-0.1733)t}$$

$$\frac{1}{100} = e^{(-0.1733)t}$$

$$\ln\left(\frac{1}{100}\right) = (-0.1733)t$$

$$t = \frac{\ln(0.01)}{-0.1733}$$

$$= \boxed{26.57 \text{ months}}$$