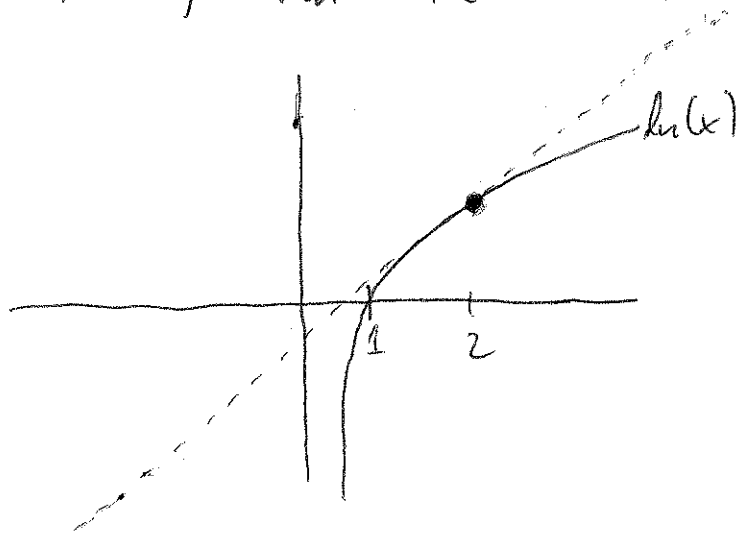


Announcements

- Hw 3, Quiz 3 reopened until 11pm
- Hw 4 due 11pm
- Quiz #4 due Monday Sept 20 11pm
- Review Session Monday
- Test #1 Wednesday Sept 22; 1.1-1.9, 2.1, 2.2

Section 2.3: Linear Approximations

Ex: Find the equation of the tangent line to the function $f(x) = \ln(x)$ at 2.



- Slope of tangent line
 $= f'(2)$

- Point on the line: $(2, \ln(2))$

Use point-slope form:

$$y - y_0 = m(x - x_0)$$

$$y - \ln(2) = m(x - 2)$$

$$\boxed{f(x) = \ln(x)}$$

- $\ln(2) \approx 0.693$

- $m \approx f'(2) \approx$ Avg rate of change of f over a small interval containing 2

$$\approx \text{Avg rate of change of } f \text{ over } [2, 2.01]$$

$$= \frac{f(2.01) - f(2)}{2.01 - 2}$$

$$= \frac{\ln(2.01) - \ln(2)}{0.01}$$

$$\approx \frac{0.698 - 0.693}{0.01} = \frac{0.005}{0.01}$$

$$= 0.5$$

- Eqn: $y - 0.693 = 0.5(x - 2)$

$$\boxed{y = 0.5(x - 2) + 0.693}$$

— Suppose your log function is broken but you need to know $\ln(2.1)$.

- Use the linear approx: $\ln(2.1) \approx 0.5(2.1 - 2) + 0.693$
 $= 0.5(0.1) + 0.693$

$$= 0.05 + 0.693$$

$$= \boxed{0.743}$$

(Actual value ≈ 0.742)

Relative Rate of Change

One quantity	Two quantities
<u>Change</u> : $(\text{new_val}) - (\text{old_val})$	Avg Rate of change: $\frac{f(b) - f(a)}{b - a}$
	instantaneous rate of change: Limit of Avg rate of change over smaller and smaller intervals
<u>Rel. Change</u> : $\frac{(\text{new_val}) - (\text{old_val})}{(\text{old_val})}$	Rel. rate of change: $\frac{f'(a)}{f(a)}$

Ex. The total output of the world's solar cells S , in megawatts, is approximately given by

$$S = 277 e^{0.368t}$$

Where t is the number of years since 2000.

(a) Estimate (using a window of size 0.01 years) the instantaneous rate of change of S in 2010.

Give units and interpret the answer.

(b) Using part (a), estimate the relative rate of change at ~~time~~ the start of 2010.

Give units and interpret the answer.

Soln: (a) Find avg rate of change of S over $[10, 10.01]$:

$$\frac{S(10.01) - S(10)}{10.01 - 10} = \dots = 4048.8 \text{ megawatts/yr}$$

[At the start of 2010, the output of the world's solar cells was increasing by 4048.8 megawatts/yr.]

$$(b) \text{ Rel rate of change} = \frac{\cancel{S'(10)}}{\cancel{S(10)}} \frac{S'(10)}{S(10)}$$

$$= \frac{4048.8 \text{ (mW/yr)}}{10982.1 \text{ (mW)}}$$

$$= 0.369 \frac{1}{\text{yr}}$$

[At the start of 2010, The output of the world's solar cells was increasing by 36.9% per year.]