

# Announcements

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- HW #3 reopened; due Fri 11pm
- HW #4 out, due Fri 11pm (See 2.1, 2.2)
- Quiz #4 at Thurs, due Mon Sept 20, 11pm
- In class Monday: Review for Test #1
- Test #1: Wed Sept 22, in class. See 1.1-1.9, 2.1, 2.2  
⇒ Bring a non-CAS calculator ←

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- $f'(a)$  = Derivative of  $f$  at  $a$   
= Slope of the tangent line to  $f$  at  $a$   
= instantaneous rate of change at  $a$   
= Avg rate of change over smaller and smaller intervals containing  $a$ . ]

Last time:  $f(x) = 2x^2$

- Avg rate of change of  $f$  over  $[1, 1+h] = 4 + 2h$
- $f'(1) = 4$

• How do we find the derivative  $f'(x)$ ? (2)

• Avg rate of change of  $f$  over  $[x, x+h]$ ?

$$\begin{aligned} \boxed{f(x)=2x^2} \quad \frac{f(b)-f(a)}{b-a} &= \frac{f(x+h)-f(x)}{(x+h)-x} \\ &= \frac{2(x+h)^2 - 2x^2}{h} \\ &= \frac{2[x^2 + 2xh + h^2] - 2x^2}{h} \\ &= \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{2x^2}}{h} \\ &= \frac{h(4x + 2h)}{h} \\ &= \boxed{4x + 2h} \end{aligned}$$

• When  $h$  is close to zero, Avg rate of change is  $4x$ .

$$\bullet \quad \boxed{f'(x) = 4x}$$

# Estimating a Derivative

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$x$	2.7	2.8	2.9	3.0	3.1
$f(x)$	-0.993	-1.030	-1.065	-1.099	-1.131

• Estimate  $f'(2.8)$ .

$$\text{(Right estimate)} \Rightarrow \frac{f(2.9) - f(2.8)}{2.9 - 2.8} = \frac{(-1.065) - (-1.030)}{0.1}$$

$$= \frac{-0.035}{0.1} = \underline{\underline{-0.35}}$$

$$\Rightarrow \frac{f(2.8) - f(2.7)}{2.8 - 2.7} = \frac{(-1.030) - (-0.993)}{\cancel{2.8} 0.1}$$

$$= -0.37$$

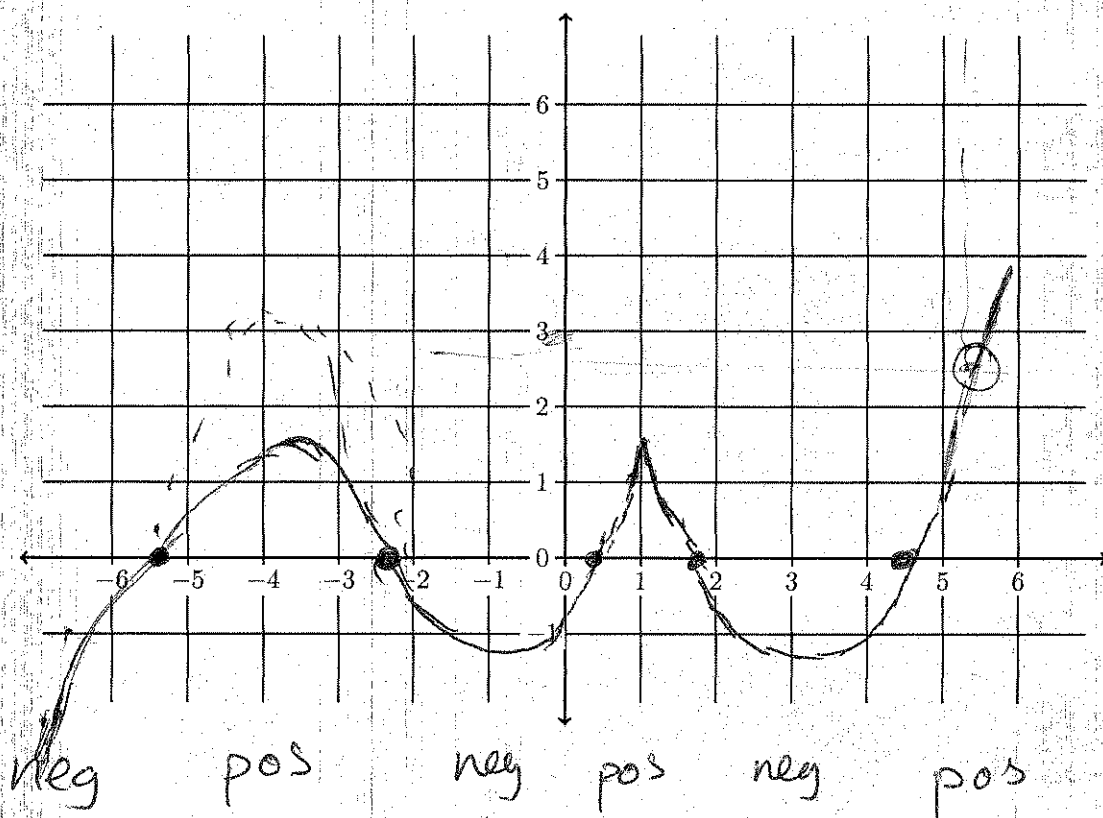
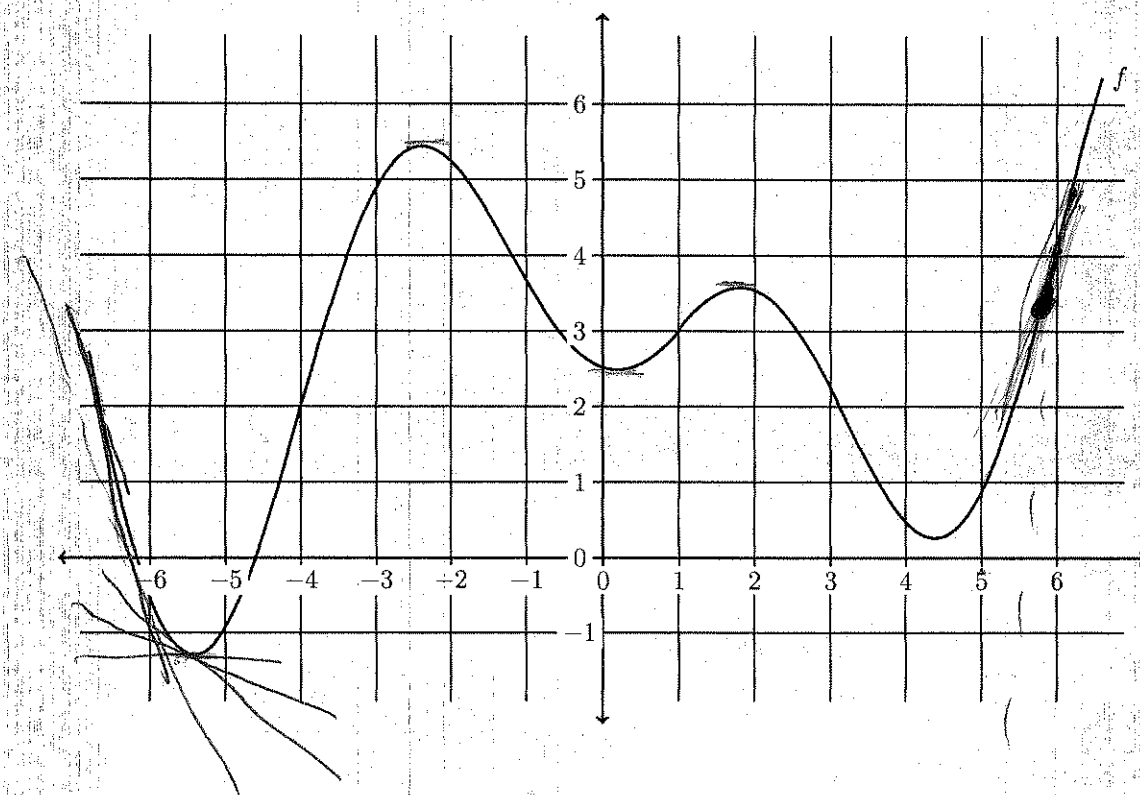
$$\text{Final Estimate} = \frac{1}{2} (\text{left Est} + \text{Right Est})$$

$$= \frac{1}{2} ((-0.37) + (-0.35)) = \boxed{-0.36}$$

## Derivatives from a graphical point of view

- $f'(x) =$  slope of tangent line to  $f$  at  $x$
- $f'(x) > 0$  for each  $x$  in  $[a, b]$ ?
  - $\Rightarrow$  slope of  $f$  at  $x > 0$  for each  $x$  in  $[a, b]$
  - $\Rightarrow f$  is increasing on  $[a, b]$
- $f'(x) < 0$  for each  $x$  in  $[a, b]$ ?
  - $\Rightarrow f$  is decreasing on  $[a, b]$
- $f'(x) = 0$  for each  $x$  in  $[a, b]$ ?
  - $\Rightarrow f$  is constant on  $[a, b]$

1. The graph of the function  $f(x)$  appears below. Sketch the derivative  $f'(x)$ .



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