1.8/1.9

Suppose you open a savings account which has a 1.5% annual interest rate, compounded annually.

In 10 years, you'll need to have \$10,000.
How much should you deposit now?

Soln $B(t)$ = Balance of the account after t years

$B(t)$: exponentially increasing function.

Discrete formula: $B(t) = B_0 (1+r)^t$

$$B(t) = B_0 (1 + 0.015)^t$$

$$B(t) = B_0 (1.015)^t$$

Want: $10000 = B_0 (1.015)^{10}$

$$B_0 = \frac{10000}{(1.015)^{10}} = 8616.67$$

Section 2.1, 2.2:

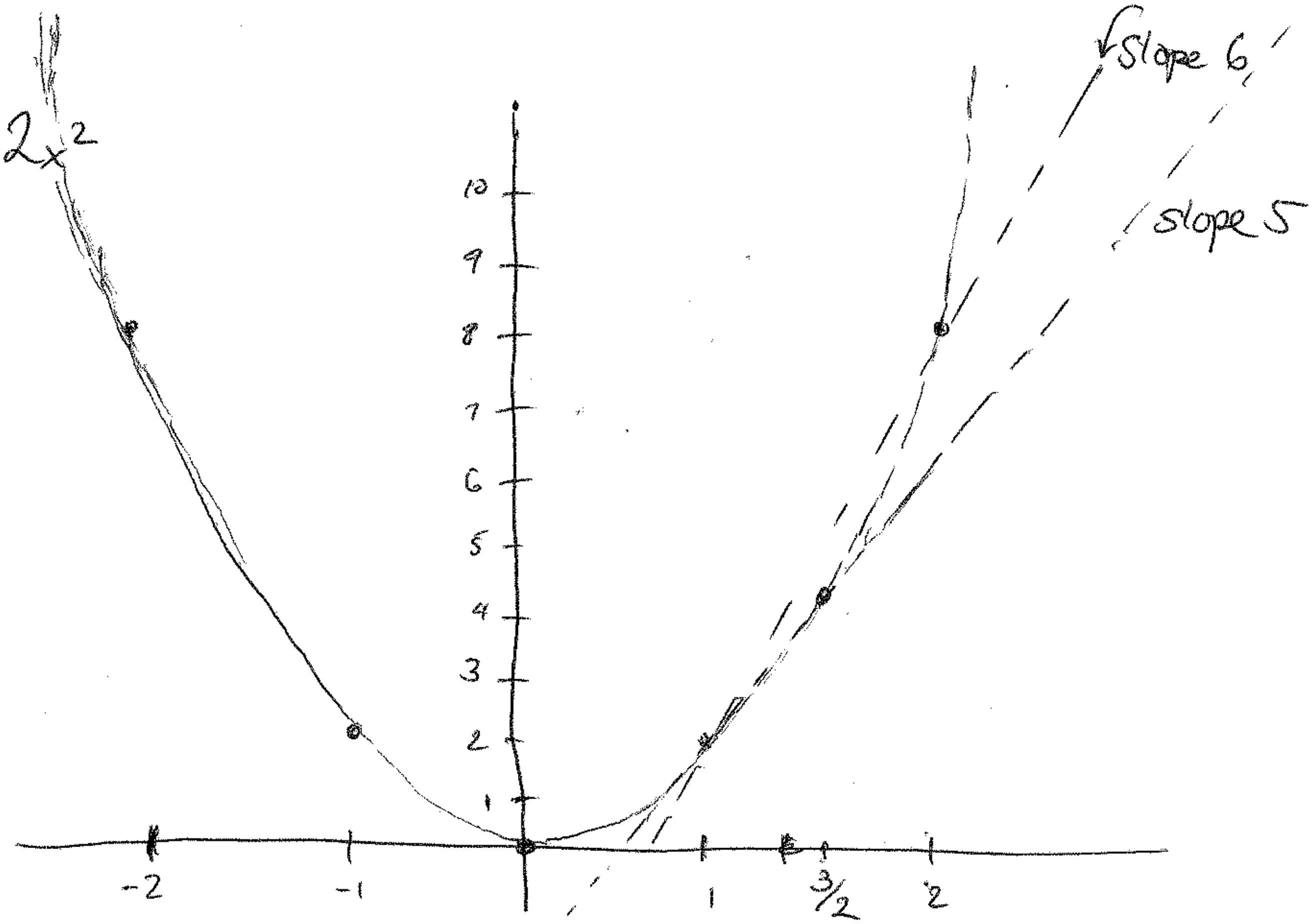
- Derivative of f at a = instantaneous rate of change of f at a

  = slope of tangent line at point a

- What is the "instantaneous rate of change"?

  $\Rightarrow$ instantaneous rate of change is the <u>limit</u> of the <u>average rate of change</u> over smaller and smaller intervals containing a.

$f(x) = 2x^2$



<u>Goal</u>: Find the derivative of f at 1; $\boxed{f'(1)}$

- First, find the avg rate of change of f on [1, 2]

$$\text{Avg. rate of change} = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{f(2) - f(1)}{2 - 1}$$

$$= \frac{2 \cdot 4 - 2}{1}$$

$f(x) = 2x^2$

$$= \frac{8 - 2}{1} = \boxed{6}$$

- Find the avg rate of change of f on $[1, \frac{3}{2}]$

$$\frac{f(b) - f(a)}{b - a} = \frac{f(\frac{3}{2}) - f(1)}{\frac{3}{2} - 1} = \frac{2(\frac{3}{2})^2 - 2}{\frac{1}{2}}$$

$$= \frac{\cancel{2} \cdot \frac{9}{\cancel{4}2} - 2}{\frac{1}{2}}$$

$$= \frac{\frac{9}{2} - 2}{\frac{1}{2}}$$

$$= \frac{\frac{5}{\cancel{2}}}{\frac{1}{\cancel{2}}} \cdot \frac{\cancel{2}}{\cancel{2}} = \boxed{5}$$

- Find the avg rate of change of f on $[1, 1+h]$

$$\frac{f(b) - f(a)}{b - a} = \frac{f(1+h) - f(1)}{(1+h) - 1} = \frac{2(1+h)^2 - 2}{h}$$

$$= \frac{2[(1+h)(1+h)] - 2}{h}$$

$$= \frac{2\left[1 + h + h + h^2\right] - 2}{h}$$

$$= \frac{2\left[1 + 2h + h^2\right] - 2}{h}$$

$$= \frac{\cancel{2} + 4h + 2h^2 \cancel{-2}}{h}$$

$$= \frac{\cancel{h}(4 + 2h)}{\cancel{h}}$$

$$= \boxed{4 + 2h} \Rightarrow$$ Instantaneous rate of change $= 4$

$f'(1) = 4.$