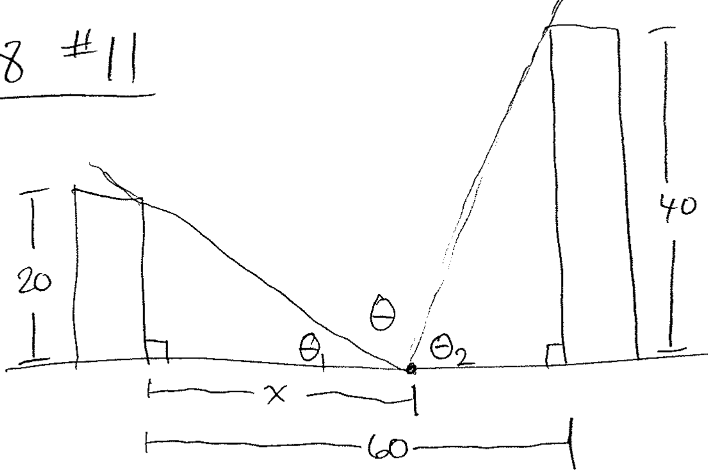
3.8 #11

(p.1)



- All distances in feet
- All times in seconds
- All angles in radians

- Know: person moving left at 4 ft/s. Therefore $\frac{dx}{dt} = -4$ ft/s.

- Next step is to write down equations relating the given quantities.

Eq 1: $\theta_1 + \theta + \theta_2 = \pi$

Eq 2: $\tan\theta_1 = \frac{20}{x}$

Eq 3: $\tan\theta_2 = \frac{40}{60-x}$

- Now lets differentiate each equation with respect to time. Note: all variables $(\theta, \theta_1, \theta_2,$ and $x)$ are functions of time.

- Diff Eq 1: $\frac{d\theta_1}{dt} + \frac{d\theta}{dt} + \frac{d\theta_2}{dt} = 0$

- Diff Eq 2: $(\sec\theta_1)^2 \frac{d\theta_1}{dt} = 20(-1)x^{-2}\cdot\frac{dx}{dt}$
$= \frac{-20}{x^2}\cdot\frac{dx}{dt}$

this is $\frac{d}{dt}(60-x)$

- Diff Eq 3: $(\sec\theta_2)^2\frac{d\theta_2}{dt} = 40(-1)(60-x)^{-2}\cdot\boxed{(-1)\cdot\frac{dx}{dt}}$

$$= \frac{40}{(60-x)^2}\cdot\frac{dx}{dt}$$

- We want to solve for $\frac{d\theta}{dt}$.

- From Diff Eq 1,

$$\frac{d\theta}{dt} = -\left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right)$$

- From Diff Eq #2 and #3:

- $$\frac{d\theta_1}{dt} = -\frac{20}{x^2(\sec\theta_1)^2}\cdot\frac{dx}{dt}$$

$$= -\frac{20(\cos\theta_1)^2}{x^2}\cdot\frac{dx}{dt}$$

- From our picture,

$$\cos\theta_1 = \frac{\text{adjacent}}{\text{hyp.}} = \frac{x}{\sqrt{x^2+20^2}}$$

$$= \frac{-20\left(\frac{x}{\sqrt{x^2+(20)^2}}\right)^2}{x^2}\cdot\frac{dx}{dt}$$

$$= -\frac{20}{x^2+(20)^2}\cdot\frac{dx}{dt}$$

- $$\frac{d\theta_2}{dt} = \frac{40}{(60-x)^2(\sec\theta_2)^2}\cdot\frac{dx}{dt}$$

$$= \frac{40(\cos\theta_2)^2}{(60-x)^2}\cdot\frac{dx}{dt}$$

- From the picture,

$$\cos\theta_2 = \frac{60-x}{\sqrt{(60-x)^2+(40)^2}}$$

$$= \frac{40\left(\frac{60-x}{\sqrt{(60-x)^2+(40)^2}}\right)^2}{(60-x)^2}\cdot\frac{dx}{dt}$$

$$= \frac{40}{(60-x)^2+(40)^2}\cdot\frac{dx}{dt}$$

• Substituting our formulas for $\frac{d\theta_1}{dt}$ and $\frac{d\theta_2}{dt}$, we get

$$\frac{d\theta}{dt} = -\left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right)$$

$$= -\left(-\frac{20}{x^2+(20)^2}\cdot\frac{dx}{dt} + \frac{40}{(60-x)^2+(40)^2}\cdot\frac{dx}{dt}\right)$$

• We want $\frac{d\theta}{dt}$ when the person is halfway; i.e. when $x=30$.
Recall $\frac{dx}{dt} = -4$.

$$\left(\frac{d\theta}{dt}\right)\Big|_{x=30} = -\left(+\frac{20}{(30)^2+(20)^2}\cdot(+4) + \frac{40}{(30)^2+(40)^2}(-4)\right)$$

$$= -(4)(20)\left(\frac{1}{(30)^2+(20)^2} - \frac{2}{(30)^2+(40)^2}\right)$$

$$= 80\left(\frac{2}{(30)^2+(40)^2} - \frac{1}{(30)^2+(20)^2}\right)$$

$$= 80\left(\frac{1}{32500}\right)$$

$$= \frac{8}{3250} \approx \boxed{0.0024615 \text{ rad/sec}}$$

(Note: The book gives an answer of -0.00246 rad/sec, but I think the sign in the book is wrong. If you see a mistake in my work, please let me know.)