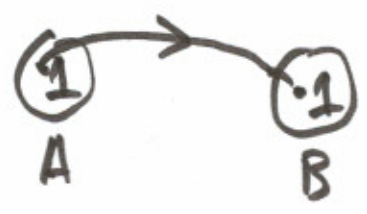
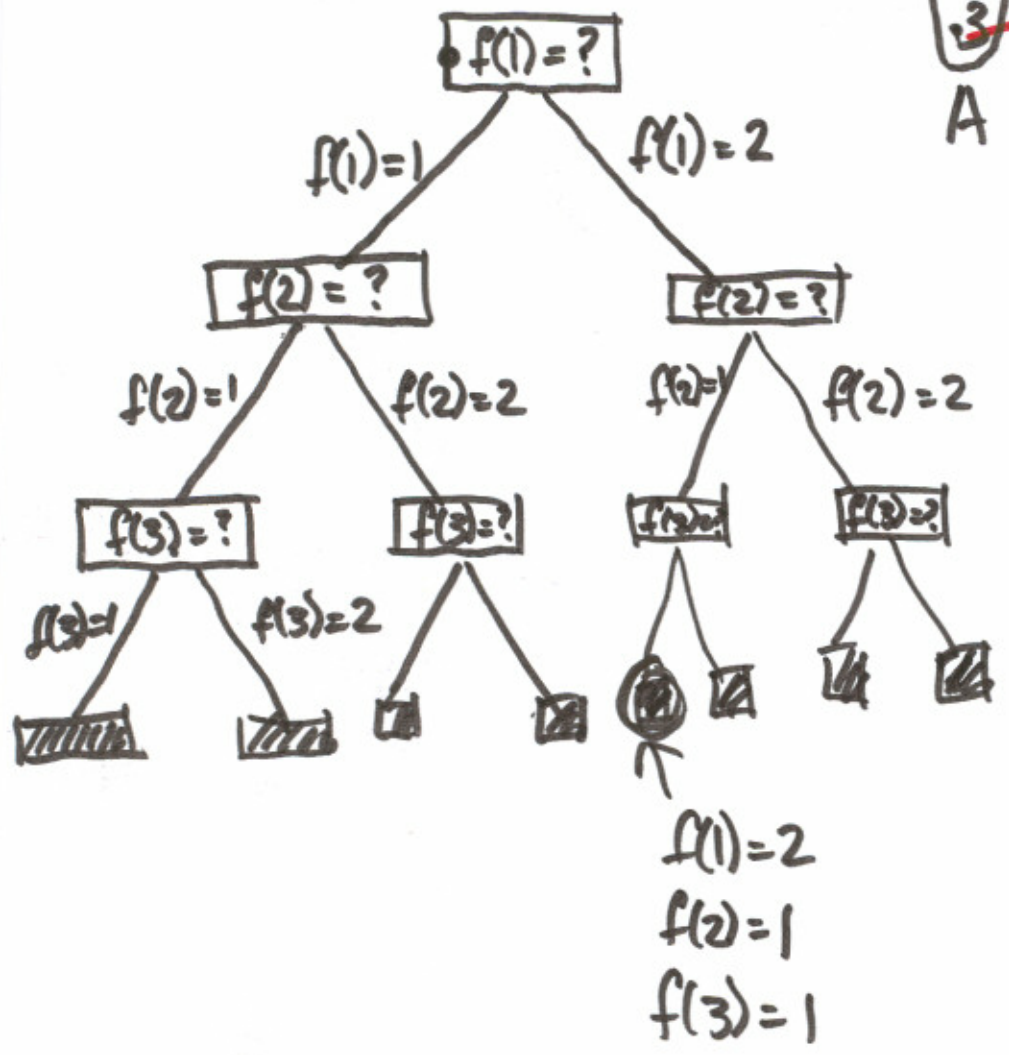
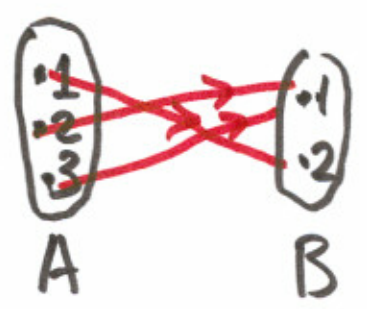


$A = \{1\}$ $B = \{1\}$

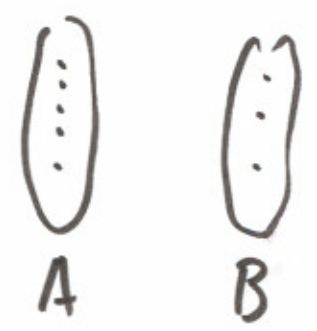
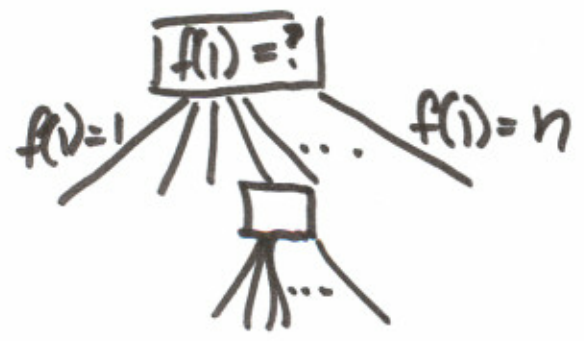


$A = \{1, 2, 3\}$ $B = \{1, 2\}$



$|\{f: A \rightarrow B\}| = 8$

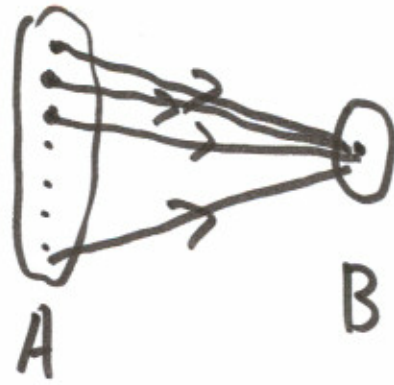
$A = \{1, 2, \dots, m\}$ $B = \{1, 2, \dots, n\}$



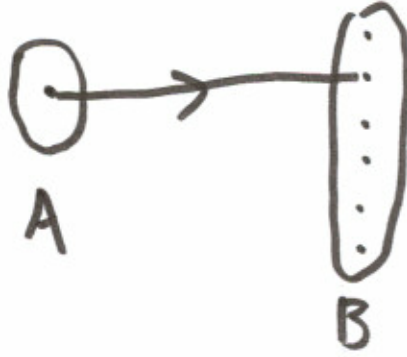
$\underbrace{n \cdot n \cdot n \dots n}_{m \text{ times}} = n^m$

$n=1$, m is large.

(2)



$m=1$, n is large



n functions

How many graphs on m vertices?



Let's add up

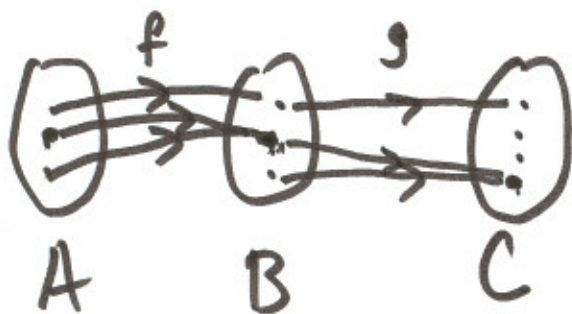
$$\begin{aligned}
 & \# \text{ graphs / w } 0 \text{ edges} - 1 \\
 & + \# \text{ graphs / w } 1 \text{ edge} - \binom{m}{1} \\
 & + \# \text{ graphs / w } 2 \text{ edges} - \binom{m}{2} \\
 & + \vdots \\
 & + \# \text{ graphs / w } \binom{m}{2} \text{ edges}
 \end{aligned}$$

Let $S = \{ \{u, v\} \mid 1 \leq u, v \leq m \}$.

$|S| = \binom{m}{2} = \alpha$

HW2: $\sum_{k=0}^{\alpha} \binom{\alpha}{k} = 2^{\alpha} = 2^{\binom{m}{2}}$

3(6):

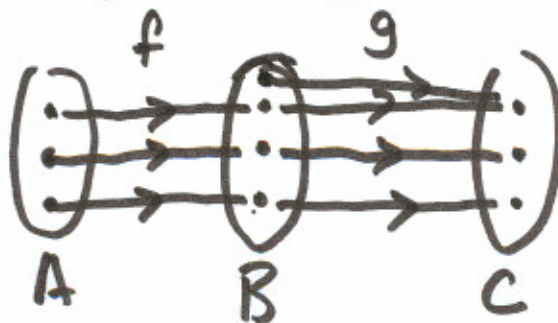


$h: A \rightarrow C$

$h(a) = g(f(a))$

If h is injective, f and g are injective.

Ans: False.



$$n=2, \mathcal{U} = \{1, 2, 3\}$$

(4)

$$\mathcal{A} = \{ \underline{\{1\}}, \underline{\{2\}}, \underline{\{3\}}, \\ \underline{\{1, 2\}}, \underline{\{1, 3\}}, \underline{\{2, 3\}}, \\ \underline{\{1, 2, 3\}} \}$$

$$\mathcal{A}_0 = \{ \underline{\{1\}} \}, \mathcal{A}_1 = \{ \underline{\{2\}}, \underline{\{1, 2\}} \}$$

$$\mathcal{A}_2 = \{ \underline{\{3\}}, \underline{\{1, 3\}}, \underline{\{2, 3\}}, \underline{\{1, 2, 3\}} \}$$

$$\mathcal{A} = \mathcal{A}_0 \cup \mathcal{A}_1 \cup \dots \cup \mathcal{A}_n$$



$$\begin{aligned} |\mathcal{A}| &= |A_0| + |A_1| + \dots + |A_n| \\ &\stackrel{||}{=} 2^{n+1} - 1 = \sum_{j=0}^n |A_j| \\ &= \sum_{j=0}^n 2^j \end{aligned}$$

What if there's only ^{or two} one \checkmark pirate?

5

$\cdot P_i$ gets all gold coins.

	weakest pirate		Strongest pirate				
$n=1$	\checkmark 1000	///	///	///	///	///	///
$n=2$	\times 0	\checkmark 1000	///	///	///	///	///
$n=3$	\checkmark 1	\times 0	\checkmark 999	///	///	///	///
$n=4$	\times 0	\checkmark 1	\times 0	\checkmark 999	///	///	///
$n=5$	\checkmark 1	\times 0	\checkmark 2	\times 0	\checkmark 998	///	///
$n=6$	\times 0	\checkmark 1	\times 0	\checkmark 1	\times 0	\checkmark 998	///