

CSTBC Homework 2

6th June 2007

1 A Bijection

Let $n \geq 1$ be an integer, let $\mathcal{U} = [n]$ (recall that $[n] = \{1, 2, \dots, n\}$), and define

$$\begin{aligned} A &= [n] \times [n-1] \times \dots \times [1] \\ B &= \{\pi \mid \pi \text{ is a permutation of } \mathcal{U}\}. \end{aligned}$$

Construct a bijection $f : A \rightarrow B$. (See hints in Lecture 2.)

2 Binomial Coefficients

By using bijections or counting the size of a set in two different ways, prove the following equalities.

1. $\sum_{k=0}^n \binom{n}{k} = 2^n$.
2. $k \binom{n}{k} = n \binom{n-1}{k-1}$.
3. $\sum_{j=1}^n j(j-1) = 2 \binom{n+1}{3}$.