Notes available at

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https://math.wvu.edu/˜kciesiel/teach/current/Calc251notesFall2019.pdf

Linear Algebra classes

Class # 1: January 14, 2020

Discussed syllabus.

Chapter 2: Matrix Algebra

Matrices: definition and the following operations

transpose, scalar multiplication, addition, and multiplication of matrices. Basic properties.

Typical exercise for this material:

Exercise 1 *For* $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 11 & -1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 0 & 8 \\ 4 & -2 \\ 5 & 1 \end{bmatrix}$ *find* A^T, $2A - 3B$, $A^T B$, *and* BA^T*:* **try at home**.

Example:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \text{ but } \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

Definition of zero matrix θ. Properties: $A + \theta = \theta + A = A$.

Definition the identity matrix I ($AI = A$ and $IB = B$) and of inverse matrix A^{-1} of a square matrix A.

Properties of multiplication:

$A(BC) = (AB)C$; $AB = I$ implies $BA = I$; however, AB need not be equal BA:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \text{ but } \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

Used for: a system $A\mathbf{x} = \mathbf{b}$ of m equations with n unknowns $x_1, \ldots, x_n$, A is $m \times n$ coefficient matrix, $\mathbf{x} = [x_1, \ldots, x_n]^T$, and $\mathbf{b} = [b_1, \ldots, b_n]^T$.

Class # 2: January 16, 2020

Chapter 4, System of linear equations $A\mathbf{x} = \mathbf{b}$:

For a system $A\mathbf{x} = \mathbf{b}$ of m equations with n unknowns $x_1, \ldots, x_n$, A is $m \times n$ coefficient matrix, $\mathbf{x} = [x_1, \ldots, x_n]^T$, and $\mathbf{b} = [b_1, \ldots, b_n]^T$.

Solving $A\mathbf{x} = \mathbf{b}$ via Gauss elimination: discuss allowable operations and format of a final matrix.

Go over Example # 1, Ch. 4, Pg. 8. (See also Example # 1 Pg. 19.)

$A\mathbf{x} = \mathbf{b}$ may have: no solutions, one solution, or infinitely many solutions.

Use of Gauss elimination, that is, using augmented matrix approach. If the system is consistent (i.e., has at least one solution), the solution must be expressed in the vertical vector form:

$$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \text{ or } \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix} \text{ or } \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}.$$

From the text: Example # 2, Ch. 4, Pg. 19
Solve exercise 2b from the Sample Test # 1, via Gauss elimination.
Solve exercise 2a from the Sample Test # 1, via Gauss elimination.
Solve problems from old Quiz #1.

Next class: Quiz # 1. Material as in Exercises 1 and 2 of the Sample Test # 1, just solved. The Sample Test # 1 will available soon at
http://www.math.wvu.edu/~kciesiel/teach/current/CurrentTeaching.html

Class # 3: January 21, 2020

If there exists a matrix B such that $BA = I$, then also $AB = I$ and B is unique. It is denoted as A^{-1} and referred to as the inverse of A. Example: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad - bc \neq 0$, then the inverse of A exists and $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Note, $A^{-1} \neq \frac{1}{A}$. In fact, the quotient $\frac{1}{A}$ has no sense at all!

A is *singular* if A^{-1} does not exist; otherwise, it is *non-singular.*

Q. What A^{-1} is useful for?

A. Many uses. E.g.: $A\mathbf{x} = \mathbf{b}$ if, and only if, $\mathbf{x} = A^{-1}\mathbf{b}$.

Also, in determining when vectors $\mathbf{b}_1, \ldots, \mathbf{b}_n \in \mathbb{R}^n$ are *linearly independent* (form a basis) — notions not discussed here, but important.

Q. When does A^{-1} exist (i.e., when A is non-singular)?

A. E.g.: when the *determinant* of A, denoted $|A|$ or $\det A$, is $\neq 0$. Calculation of the determinants to be discussed, chapter 7.

Q. When A is non-singular, how to find A^{-1}?

A. Gaussian elimination (again), to be explained.

Finding A^{-1} via Gaussian elimination: Chapter 9. To find A^{-1}: (1) write augmented matrix $[A; I]$; (2) Gaussian elimination to transform it to a matrix $[I; B]$; (3) declare that A^{-1} equals B.

Go over Exercises 4, 5 from the sample test. Possibly also Example 1, Ch. 9, Pg 5 (or read at home).

Calculation of the determinant: Via arbitrary row (or column) expansion (known as Laplace Expansion Method), definition (**not in the "textbook"**), Example on page Ch. 7, Pg 4. Take a look at Theorem Ch. 7, Pg 2, the properties of the determinant – leads to Gaussian elimination. Solve (the same problem) using Gaussian elimination, see Ch. 7, Pg 6. Solve problem 3 from the Sample Test # 1.

Solving $A\mathrm{x} = \mathbf{b}$ via *Cramer Rule*: application of determinants. Just state (Ch. 6, Pg 7), no exercises.