Set Theory, Math 783, Spring 2016:
Notes and homework assignments

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Class of January 12:

Handed and discussed course syllabus.

Presented the solutions of the following problems related to Final Test of Fall 2015:

Ex. Show that there exists a function $f\colon\mathbb{R}\to\mathbb{R}$ with the property that $|\{x\in\mathbb{R}\colon f(x)=g(x)\}|<\mathfrak{c}$ for every continuous $g\colon\mathbb{R}\to\mathbb{R}$.

Solution: Recall that the class $\mathcal{C}(\mathbb{R})$ of continuous functions $g\colon\mathbb{R}\to\mathbb{R}$ has cardinality continuum. Let $\{g_\xi\colon\xi<\mathfrak{c}\}$ be an enumeration of $\mathcal{C}(\mathbb{R})$ and let $\{r_\xi\colon\xi<\mathfrak{c}\}$ be an enumeration of $\mathbb{R}$ (with no repetitions).

For every $\xi<\mathfrak{c}$ choose $f(r_\xi)\in\mathbb{R}\setminus\{g_\zeta(r_\xi)\colon\zeta<\xi\}$. (The choice can be made, since $|\{g_\zeta(r_\xi)\colon\zeta<\xi\}|\le|\xi|<\mathfrak{c}=|\mathbb{R}|$.) Then f is defined on every real number and it as desired. Indeed, for every $g\in\mathcal{C}(\mathbb{R})$ there is a $\xi<\mathfrak{c}$ such that $g=g_\xi$, and so the set

$$\{x\in\mathbb{R}\colon f(x)=g(x)\}=\{r\in\mathbb{R}\colon f(r)=g_\xi(r)\}\subset\{r_\zeta\colon\zeta<\xi\}$$

has cardinality $\le|\xi|<\mathfrak{c}$. ∎

This is similar to:

Ex. Let κ be an infinite regular cardinal. Show that for every family $\mathcal{F}\subset\kappa^\kappa$ with $|\mathcal{F}|\le\kappa$ there exists a function $h\colon\kappa\to\kappa$ such that for every $f\in\mathcal{F}$

$$|\{\alpha<\kappa\colon h(\alpha)\le f(\alpha)\}|<\kappa.$$

Solution (sketch): Let $\mathcal{F}=\{f_\xi\colon\xi<\kappa\}$. For every $\alpha<\mathfrak{c}$ choose

$$h(\alpha)\in\kappa\setminus\bigcup_{\xi\le\alpha}(f_\xi(\alpha)+1)=\kappa\setminus\bigcup_{\xi\le\alpha}\{\eta\colon\eta\le f_\xi(\alpha)\},$$

so that $h(\alpha)>f_\xi(\alpha)$ for every $\xi\le\alpha$.

The choice can be made since, by regularity of κ, $\bigcup_{\xi\leq\alpha}(f_\xi(\alpha)+1)$ is bounded in κ. Then, for every $f_\xi \in \mathcal{F}$ the set

$$\{\alpha<\kappa : h(\alpha)\leq f_\xi(\alpha)\}\subset\{\alpha<\kappa:\alpha<\xi\}=\xi$$

has cardinality $<\kappa$, i.e., h is as needed. ∎

(We noted, that this is of interest even when $\kappa=\omega$.)

Ex. Construct an $f\colon\mathbb{R}\to\mathbb{R}$ such that $f^{-1}(r)$ is a Bernstein set for every $r\in\mathbb{R}$.

Solution: Let $\mathcal{P}$ be that family of all perfect subsets of $\mathbb{R}$. Then $|\mathcal{P}\times\mathbb{R}|=\mathfrak{c}$, so we can choose an enumeration $\{\langle P_\xi, r_\xi\rangle : \xi<\mathfrak{c}\}$ of $\mathcal{P}\times\mathbb{R}$.

By induction, choose a sequence $\{x_\xi : \xi<\mathfrak{c}\}$ such that

- $x_\xi \in P_\xi \setminus \{x_\zeta : \zeta<\xi\}$.

The choice can be made, since $|\{x_\zeta : \zeta<\xi\}|\leq|\xi|<\mathfrak{c}=|P_\xi|$.

Since $x_\zeta\neq x_\xi$ for every $\zeta<\xi<\mathfrak{c}$, we can define f on $\{x_\xi : \xi<\mathfrak{c}\}$ by putting $f(x_\xi)=r_\xi$ for every $\xi<\mathfrak{c}$ and extending it in an arbitrary way on $\mathbb{R}\setminus\{x_\xi : \xi<\mathfrak{c}\}$.

To see that f is as desired, notice that $f^{-1}(r)\cap P\neq\emptyset$ for every $r\in\mathbb{R}$ and $P\in\mathcal{P}$. Indeed, if $\xi<\mathfrak{c}$ is such that $\langle P_\xi, r_\xi\rangle=\langle P, r\rangle$, then $x_\xi\in f^{-1}(r_\xi)\cap P_\xi = f^{-1}(r)\cap P$.

Thus, for every $r\in\mathbb{R}$, $f^{-1}(r)$ is a Bernstein set, since both $f^{-1}(r)$ and its complement (which contain $f^{-1}(s)$ for any $s\neq r$) intersect every $P\in\mathcal{P}$.

Class of January 14:

Review:

- We will study the Axiomatic Set Theory
  - Take a look (at home) at the ZFC axioms from Appendix A.
  - Recall, that there always will exist statement independent of the axioms.
- Definitions of: *well-ordered set*, *order isomorphism*, *initial segment*, and of $O(x_0)$.
- Theorems 4.1.5: *Every proper initial segment S of a well-ordered set W is of the form $O(\xi)$ for some $\xi \in W$.*
- Theorems 4.1.6: (Principle of transfinite induction) *If a set A is well-ordered, $B \subset A$, and for every $x \in A$ the set B satisfies the condition*

$$O(x) \subset B \Rightarrow x \in B, \tag{1}$$

  then $B = A$.
- Sec 4.2: Ordinal numbers the in von Neumann form: sets of their predecessors.
- Sec 4.3: Definition by transfinite induction: big part of this semester study.
- (Well-ordering or Zermelo's theorem) *Every nonempty set X can be well ordered.*
- Sec 5.1: Carinal numbers as initial ordinals. Cantor's Theorem.
- Sec 5.2: Cardinal arithmetic.

  Cor 5.2.5: *If λ and κ are infinite cardinals then*

$$\kappa \oplus \lambda = \kappa \otimes \lambda = \max\{\kappa, \lambda\}.$$

  Thm 5.2.11: $|\mathbb{R}| = |2^\omega| = |\mathcal{P}(\omega)| = \mathfrak{c}$.

Read at home *Exercise 4 page 73: If κ is an infinite cardinal and $|X_\alpha| \le \kappa$ for all $\alpha < \kappa$, then $\left|\bigcup_{\alpha<\kappa} X_\alpha\right| \le \kappa$.*

SOLUTION. If $\bigcup_{\alpha<\kappa} X_\alpha = \emptyset$, then clearly the inequality holds. So, assume that $\bigcup_{\alpha<\kappa} X_\alpha \neq \emptyset$ and choose $x \in \bigcup_{\alpha<\kappa} X_\alpha$. For every $\alpha < \kappa$ let $X^*_\alpha = X_\alpha$ if $X_\alpha \neq \emptyset$ and $X_\alpha = \{x\}$, otherwise. Then $\bigcup_{\alpha<\kappa} X_\alpha = \bigcup_{\alpha<\kappa} X^*_\alpha$ and $|X^*_\alpha| \le \kappa$ for all $\alpha < \kappa$.

For every $\alpha < \kappa$ let f_α be a surjection from κ onto X^*_α. (We needed $X^*_\alpha \neq \emptyset$ to ensure existence of f_α.) Then function $F\colon \kappa \times \kappa \to \bigcup_{\alpha<\kappa} X^*_\alpha$, $F(\alpha,\beta) = f_\alpha(\beta)$ is a surjection. Therefore $\left|\bigcup_{\alpha<\kappa} X_\alpha\right| = \left|\bigcup_{\alpha<\kappa} X^*_\alpha\right| \le |\kappa \times \kappa| = \kappa$, as required. ∎

Ex. For an arbitrary set Γ let

$$c_0(\Gamma) = \{f \in \mathbb{R}^\Gamma \colon \{\gamma \in \Gamma \colon |f(\gamma)| > \varepsilon\} \text{ is finite for every } \varepsilon > 0\}$$

and

$$c_{00}(\Gamma) = \{f \in \mathbb{R}^\Gamma \colon \{\gamma \in \Gamma \colon |f(\gamma)| \neq 0\} \text{ is finite}\}.$$

Find the cardinalities of the following sets: $c_0(\omega)$, $c_0(\omega_1)$, $c_0(\mathfrak{c}^+)$, $c_{00}(\omega)$, $c_{00}(\omega_1)$, and $c_{00}(\mathfrak{c}^+)$.

SOLUTION. We will show that for every cardinal $\kappa \ge \omega$:

$$\max\{\kappa, \mathfrak{c}\} \le |c_{00}(\kappa)| \le |c_0(\kappa)| \le \kappa^\omega.$$

Since, for $\omega \le \kappa \le \mathfrak{c}$ we have $\max\{\kappa,\mathfrak{c}\} = \kappa^\omega = \mathfrak{c}$, the inequalities imply that $|c_{00}(\omega)| = |c_0(\omega)| = |c_{00}(\omega_1)| = |c_0(\omega_1)| = \mathfrak{c}$. Similarly, the equation $\max\{\mathfrak{c}^+,\mathfrak{c}\} = (\mathfrak{c}^+)^\omega = \mathfrak{c}^+$ and the inequalities imply that $|c_{00}(\mathfrak{c}^+)| = |c_0(\mathfrak{c}^+)| = \mathfrak{c}^+$.

The inequality $\kappa \le |c_{00}(\kappa)|$ follows from inclusion $\{\chi_{\{\xi\}} \colon \xi < \kappa\} \subset c_{00}(\kappa)$, as clearly $|\{\chi_{\{\xi\}} \colon \xi < \kappa\}| = \kappa$.

$\mathfrak{c} \le |c_{00}(\kappa)|$ follows from inclusion $\{r\chi_{\{0\}} \colon r \in \mathbb{R}\} \subset c_{00}(\kappa)$, as clearly $|\{r\chi_{\{0\}} \colon r \in \mathbb{R}\}| = \mathfrak{c}$.

$|c_{00}(\kappa)| \le |c_0(\kappa)|$ is justified by inclusion $c_{00}(\kappa) \subset c_0(\kappa)$.

To see $|c_0(\kappa)| \le \kappa^\omega$, for every $f \in \mathbb{R}^\Gamma$ let $\operatorname{supp}(f) = \{\gamma \in \Gamma \colon f(\gamma) \neq 0\}$ and notice that for every $f \in c_0(\kappa)$ the set $\operatorname{supp}(f) = \bigcup_{n=1}^\infty \{\gamma \in \Gamma \colon |f(\gamma)| > \frac{1}{n}\}$ is at most countable, as a countable union of finite sets. Since the mapping $c_0(\kappa) \ni f \mapsto f \restriction \operatorname{supp}(f) \in \bigcup_{S\in[\kappa]^{\le\omega}} \mathbb{R}^S$ is injective and $|\mathbb{R}^S| = (2^\omega)^{|S|} \le \mathfrak{c}$, we get $|c_0(\kappa)| \le \left|\bigcup_{S\in[\kappa]^{\le\omega}} \mathbb{R}^S\right| \le [\kappa]^{\le\omega}| \otimes \mathfrak{c} = \kappa^\omega \otimes 2^\omega = \kappa^\omega$, as needed.

Class of January 19:

Review of Section 5.3: Cofinality

- A *cofinality* $\text{cof}(\alpha)$ of a limit ordinal number α is the smallest β for which there exists a map $f\colon \beta \to \alpha$ *coffinal* in α, that is, such that $\alpha = \bigcup_{\xi<\beta} f(\xi)$. (You can define also $\text{cof}(\alpha + 1) = 1$.)
- There is strictly increasing coffinal map $f\colon \text{cof}(\alpha) \to \alpha$.
- $\text{cof}(\alpha)$ is always a cardinal number.
- A limit ordinal number α is *regular* provided $\alpha = \text{cof}(\alpha)$. In particular, every regular limit ordinal number is a cardinal number.
- $\text{cof}(\alpha)$ is a regular ordinal (and cardinal): $\text{cof}(\text{cof}(\alpha)) = \text{cof}(\alpha)$.
- ω is a regular cardinal.
- κ^+ is a regular cardinal for every infinite cardinal κ.
- If $A \subset \alpha$ and $|A| < \text{cof}(\alpha)$, then $A \subset \beta$ for some $\beta < \alpha$, that is, A is bounded in α.
- $\kappa^{\text{cof}(\kappa)} > \kappa$ for every infinite cardinal κ.
- $\text{cof}(\mathfrak{c}) > \omega$..

Solve the remaining two exercises from the Fall 2015 final test

Ex. 1. Show that $|S| = 2^\kappa$, where $S = \{f \in \kappa^\mu\colon |\{\gamma \in \mu\colon f(\gamma) \neq 0\}| \leq \kappa\}$ while $\mu = \kappa^+$ and κ is an infinite cardinal.

Solution: For every $\alpha < \mu$ let $S_\alpha = \{f \in \kappa^\mu\colon \{\gamma \in \mu\colon f(\gamma) \neq 0\} \subset \alpha\}$. Since $\mu = \kappa^+$ is a regular cardinal, for every $f \in S$ there is an $\alpha < \mu$ such that $\{\gamma \in \kappa\colon f(\gamma) \neq 0\} \subset \alpha$, that is, $f \in S_\alpha$. So, $S = \bigcup_{\alpha<\mu} S_\alpha$. Also, the map $S_\alpha \ni f \mapsto f \restriction \alpha \in \kappa^\alpha$ is a bijection. So, $|S_\alpha| = |\kappa^\alpha| = \kappa^{|\alpha|} \leq \kappa^\kappa = 2^\kappa$. Hence, S is a union of $\kappa^+ \leq 2^\kappa$ sets, each of cardinality $\leq 2^\kappa$. So, $|S| \leq 2^\kappa$.

On the other hand, $|S| \geq |S_\kappa| = |\kappa^\kappa| = 2^\kappa$. Thus, $|S| = 2^\kappa$.

Ex. 3. Construct Bernstein set $B \subset \mathbb{R}$ with $B + B = \mathbb{R}$. (Recall, that a Bernstein set is not Lebesque-measurable and has no Baire property.)

Solution: Let $\{P_\xi : \xi < \mathfrak{c}\}$ be an enumeration of all perfect subsets of $\mathbb{R}$ and let $\{r_\xi : \xi < \mathfrak{c}\}$ be an enumeration of $\mathbb{R}$.

Aiming for $B = \{b_\xi : \xi < \mathfrak{c}\} \cup \{r_\xi - b_\xi : \xi < \mathfrak{c}\}$, by transfinite induction on $\xi < \mathfrak{c}$ we define the sequences $\langle a_\xi : \xi < \mathfrak{c}\rangle$ and $\langle b_\xi : \xi < \mathfrak{c}\rangle$ by choosing in step $\xi < \mathfrak{c}$:

(I_ξ) $b_\xi \in P_\xi \setminus D_\xi$, where $D_\xi = \{a_\zeta : \zeta < \xi\} \cup \{r_\xi - a_\zeta : \zeta < \xi\}$;

(J_ξ) $a_\xi \in P_\xi \setminus E_\xi$, where $E_\xi = \{b_\zeta : \zeta \le \xi\} \cup \{r_\zeta - b_\zeta : \zeta \le \xi\}$.

This can be done since $|P_\xi| = \mathfrak{c}$, while $|E_\xi| \le |\xi + 1| \oplus |\xi + 1| < \mathfrak{c}$ and $|D_\xi| \le |\xi| \oplus |\xi| < \mathfrak{c}$. This finishes the inductive construction of the sequences.

The purpose of sets D_ξ and E_ξ is to ensure that the set $A = \{a_\xi : \xi < \mathfrak{c}\}$ is disjoint with B. Indeed, this is the case, since for any $\xi, \zeta < \mathfrak{c}$ we have $a_\xi \neq b_\zeta$ and $a_\xi \neq r_\zeta - b_\zeta$:

- this is guaranteed by (J_ξ) when $\xi \ge \zeta$, and
- by (I_ζ) when $\zeta > \xi$ (as $b_\zeta \neq r_\zeta - a_\xi$ implies $a_\xi \neq r_\zeta - b_\zeta$).

Now, B is a Bernstein set since for every perfect set P:

- $P \not\subset B$, as $a_\xi \in P \cap A \subset P \cap (\mathbb{R} \setminus B)$, where $\xi < \mathfrak{c}$ is such that $P = P_\xi$;
- $P \not\subset \mathbb{R} \setminus B$, as $b_\xi \in P \cap B$, where $\xi < \mathfrak{c}$ is such that $P = P_\xi$.

Finally, $B + B = \mathbb{R}$, since for every $r \in \mathbb{R}$ there $\xi < \mathfrak{c}$ for which $r = r_\xi$ and so, $r = r_\xi$ is a sum of $b_\xi, r_\xi - b_\xi \in B$.

Next class, we will go to new material: Chapter 7

Class of January 21:

Started Section 7.2: Discussed material up to Theorem 7.2.5. The proof of this theorem was sketched.

Class of January 26:

Finish the proof of Theorem 7.2.5.
Discuss briefly Exercises 1 and 3.
Go over Section 7.1.

Written assignment for February 2: Prove the following theorem of Sierpiński and Zygmund:[1]

There exists a function $f: \mathbb{R} \to \mathbb{R}$, known as Sierpiński-Zygmund function*, such $f \restriction X$ is discontinuous for every $X \in [\mathbb{R}]^{\mathfrak{c}}$.*

Hint: *Use (you do not need to prove this) the fact that every continuous function $h: X \to \mathbb{R}$, with $X \subset \mathbb{R}$, has a continuous extension $\bar{h}: G \to \mathbb{R}$ to a G_δ set $G \subset \mathbb{R}$. (This can be proved by noticing that the set of all $z \in \mathbb{R}$ for which the limit $\lim_{x \to z} h(x)$ does not exist is an F_σ set.) Noticing this extension result, in the content of this theorem, is the key in the Sierpiński-Zygmund result.*

Class of January 28:

Started Section 7.3: Discussed material up to Corollary 7.3.7. Started (sketches) the proof of Theorem 7.3.8. So far we proved that

- If a function $f: \mathbb{R} \to \mathbb{R}$ has a disconnected graph, then there exists a closed set $P \subset \mathbb{R}^2$ disjoint with (the graph of) f and such that its projection $p[P]$ contains a non-trivial interval.

Class of February 2:

Finish Section 7.3: prove (finish) Theorem 7.3.8 and Theorem 7.3.9 (including lemmas).

Lemmas left for the next class.

[1]This is the correction of Exercise 5, p. 111. The statement of this exercise is true, but requires no hint. If you cannot prove the general statement, try first prove its version from Exercise 5 (without using the hint).

Class of February 4:

Finish the proof of Theorem 7.3.9, including Lemmas 7.3.10 and 7.3.11.

Start Chapter 8. (We will skip section 7.4.)

Written assignment for Thursday, February 18. (I may consider this as a take-home Mid Term Test.) Solve Exercise 3 page 117. Keep the solutions of parts (a) and (b) separate—they will be graded separately.

Hint: For part (a), construct V by a transfinite induction.

For part (b) start with a Hamel H basis which is a Bernstein set. It might be convenient to use one that contains 1 (i.e., with $1 \in H$). If used, explain why such an H exist.

On Tuesday, February 9, Prof. Wojciechowski will give a lecture. (Cancelled due to bad weather.)
There will be no class on Thursday, February 11.

Class of February 16:

Covered section 8.1 up to, including, the proof of Rasiowa-Sikosrki lemma.

Class of February 18:

Collect assignment.

Continue covering section 8.1: covered up to statement of Theorem 8.1.5 and proved Lemma 8.1.8.

Class of February 23:

Finish section 8.1 by restating Theorem 8.1.5, proving Lemma 8.1.9, and proving the theorem.

Start going over section 8.2, on Martin's Axiom. (Covered up to, including Theorem 8.2.1.)

Assignment alternative to one of the party of previous assignment. Solve Exercise 1 page 138. You need to use Rasiowa-Sikorski Lemma (and the language of partial ordered set) to get a credit.

Class of February 25:

Restate Theorem 8.1.5 on the consistency of the Martin's Axiom, MA, and $\neg$CH, including the definition of a ccc PO set.

Go over the consequences of MA, starring with Theorem 8.2.2 on the existence of scale.

Went also over a big part of the proof of Theorem 8.2.3. Concluded Corollaries 8.2.4 and 8.2.5.

Class of March 1:

Hand the solutions of Exercise 3 page 117: assigned on February 18.

Also briefly discuss the solution of part (b) of this problem.

Finish the proof of Theorem 8.2.3.

Time permitting, go over Theorem 8.2.6. (Not covered.)

Solutions of the assignment of February 4, for February 18: Exercise 3, p. 117.

Ex. 3 page 117: Construct a Vitali et V such that:

(a) $V + V = \mathbb{R}$;

(b) $V + V$ *is a Bernstein set.*

Solution: Part (a): Let $\mathbb{R} = \{x_\xi \in \mathbb{R} \colon \xi < \mathfrak{c}\}$. We will choose, by induction on $\xi < \mathfrak{c}$, the sequence $\langle v_\xi \in \mathbb{R} \colon \xi < \mathfrak{c} \rangle$ aiming for V being an extension of the sets $V_\xi = \bigcup_{\zeta<\xi}\{v_\zeta, x_\zeta - v_\zeta\}$. This will guarantee that $V+V = \mathbb{R}$, as for every $x \in \mathbb{R}$ there is $\xi < \mathfrak{c}$ such that $x = x_\xi = v_\xi + (x_\xi - v_\xi) \in V_{\xi+1} + V_{\xi+1} \subset V + V$. The trick is to ensure that no two elements of V_ξ are in the $\sim$ relation.

For this, choose $v_\xi \in \mathbb{R}$ outside of the following set, of cardinality $< \mathfrak{c}$:

$$(\mathbb{Q} + V_\xi) \cup [x_\xi - (\mathbb{Q} + V_\xi)] \cup (x_\xi/2 + \mathbb{Q}).$$

The first set, $\mathbb{Q} + V_\xi$, ensures that $v_\xi \not\sim v$ for every $v \in V_\xi$. The second set, $x_\xi - (\mathbb{Q} + V_\xi)$, ensures that $x_\xi - v_\xi \not\sim v$ for every $v \in V_\xi$. While the third set, $x_\xi/2 + \mathbb{Q}$, ensures that $x_\xi - v_\xi \not\sim v_\xi$.

Thus, the construction ensures that no two distinct elements of $V_\mathfrak{c}$ are in the $\sim$ relation. Finally, we can extend V to a full Vitali set V, which will not affect that fact that $V + V = V_\mathfrak{c} + V_\mathfrak{c} = \mathbb{R}$.

Part (b): Let H be a Hamel basis from Corollary 7.3.7, that is, such that $|H \cap P| = \mathfrak{c}$ for every perfect set P.

First, notice that we can assume that $1 \in H$. Indeed, there exist distinct $x_1, \ldots, x_n \in H$ and nonzero numbers $q_1, \ldots, q_n \in \mathbb{Q}$ with the property that $1 = q_1 x_1 + \cdots + q_n x_n$. Then $(H \setminus \{x_0\}) \cup \{1\}$ is the Hamel basis with the desired properties.

Now, assuming that indeed $1 \in H$, notice that $V = \mathrm{LIN}_\mathbb{Q}(H \setminus \{1\})$ is a desired Vitali set.

Indeed, it is a Vitali set, since for every every $x \in \mathbb{R}$ there exist unique numbers $v \in V$ and $q \in \mathbb{Q}$ such that $x = v + q \cdot 1$. (This follows from the uniqueness of a representation of each number in a Hamel basis.)

To see that V is a Bernstein set, notice that $V + V = V$. Clearly, V intersects every perfect set, since so does $H \setminus \{1\} \subset V$. Also, $\mathbb{R} \setminus V$ intersects every perfect set, since so does its subset $1 + V$, being a translation of a set with such property. ■

Class of March 3:

State Theorem 8.2.6. Time permitting, it will be proved later in the semester.

State and prove Theorem 8.2.7.

Time permitting, start discussing Theorem 8.2.8.

Homework assignments. Start working on Exercises 2 and 3, page 153.

Class of March 8:

State and prove Theorem 8.2.8.

Class of March 10:

Discuss a bit a philosophy of why we use properties like MA or CH.

Go over some additional details of the proof of Theorem 8.2.8,

State and prove Corollaries 8.2.9 and 8.2.10.

Class of March 15:

State and prove Lemma 8.2.11.

Start proving Theorem 8.2.12: reduced transfinite induction requirements to the main combinatorial task.

Class of March 17:

Finish proving Theorem 8.2.12.

After Spring break: move to the next section.

Class of March 29:

Section 8.3. Introduction to Suslin Hypothesis (SH).
Prove of Theorem 8.3.1, that MA+not CH implies (SH).

Class of March 31:

Went over Lemma 8.3.2.
Stated Diamond Principle $\diamondsuit$ and proved Proposition 8.3.5.
Still no definition of closed unbounded sets club.

Class of April 5:

Define closed unbounded sets, club, and stationary sets.
Go over Propositions 8.3.3 and 8.3.4.
Go over Lemma 8.3.6.

Written assignment for Tuesday, April 12. Solve Exercise 4 page 162.

Class of April 7:

Possibly, solve in class Exercises 5 and 6, page 162.
Start the proof of Theorem 8.3.7.