

MATH 251  
Instr. K. Ciesielski  
Spring 2016

**SAMPLE TEST # 3**

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

**Ex. 1.** Show that the following limit does not exist

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3y}{x^4 + 7y^4}$$

**Ex. 2.** Compute the first order partial derivatives of  $f(x, y, z) = ze^{x^2} \cos y$ .

**Ex. 3.** Compute all second order partial derivatives of  $g(s, t) = e^{5t} + t \sin(3s)$ .

**Ex. 4.** Find an equation of the plane tangent to the surface  $z = x^2 - 5y^3$  at the point  $P(2, 1, -1)$ .

**Ex. 5.** Find the absolute maximum and the absolute minimum of the function  $f(x, y) = x^3 - xy$  on the region bounded below by parabola  $y = x^2 - 1$  and above by line  $y = 3$ . You will get credit **only** if **all** critical points are found.

**Ex. 6.** Find the volume of the solid bounded above by the surface  $z = 28xy$ , bounded below by  $xy$ -plane, and which is above the region bounded by  $y = x^6$  and  $y = x$ .

**Ex. 7.** Evaluate  $\int_0^1 \int_0^x 4e^{x^2} dy dx$

**Ex. 8.** Find the point on the cone  $z = \sqrt{x^2 + y^2}$  which is the closest to the point  $(4, -8, 0)$ .

**Ex. 9.** Find the directional derivative of  $f(x, y) = 10e^y \sin x$  at the point  $P(\pi/4, 0)$  in the direction of the vector  $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$ .

**Ex. 10.** Find the gradient of  $g(x, y, z) = x^2 + e^{yz} + \cos(x + 2y)$ .