MATH 251 Instr. K. Ciesielski Spring 2014

## SAMPLE TEST # 4

Solve the following exercises. Show your work.

- Ex. 1. Set up the integral formulas, including the limits of the integrations, for the following problems. Do not evaluate the integrals! Where appropriate, use polar, cylindrical, or spherical coordinates.
  - (a) The volume of the solid bounded by  $z = x^2 + y^2$ , z = 0, x = 0, y = 0, and x + y = 1.
  - (b) The mass of the plane lamina bounded by  $y=x^2$  and y=2x+3, with the density  $\delta(x,y)=x^2$ .
  - (c) The mass of the solid T with the density  $\delta(x, y, z) = x^2 + e^z$  bounded by the surfaces: 6x + 2y + z = 12, x = 0, y = 0, and z = 0.
- Ex. 2. Evaluate the integrals:

(a) 
$$\int_0^1 \int_0^{\pi} \frac{1}{x+1} + \sin y \, dy \, dx =$$

(b) 
$$\int_{-2}^{0} \int_{0}^{y} (x + 2y^{2}) dx dy =$$

- (c)  $\int \int_R \frac{dy \ dx}{\sqrt{9-x^2-y^2}}$ , where R is the second quadrant region bounded by  $x^2+y^2=4$ .
- **Ex. 3.** Find the mass of the solid bounded by the hemisphere  $x^2 + y^2 + z^2 \le R^2$ ,  $z \ge 0$ , with the density  $\delta(x, y, z) = x^2 + y^2 + z^2$ .
- **Ex. 4.** Find the mass of the plane lamina bounded by x = 0 and  $x = 9 y^2$  with density  $\delta(x, y) = x^2$ .
- **Ex. 5.** Evaluate  $\int_C xy \, ds$ , where C is the parametric curve for which  $x=3t, \ y=t^4$ , and  $0 \le t \le 1$ .
- **Ex. 6.** Evaluate the integral, where C is the graph of  $y = x^3$  from (-1, -1) to (1, 1)

$$\int_C y^2 dx + x dy =$$

Ex. 7. Determine if the following vector field is conservative. Find potential function for a field, if it is conservative.

1

(a) 
$$\mathbf{F} = \left(x^3 + \frac{y}{x}\right)\mathbf{i} + (y^2 + \ln x)\mathbf{j}$$

(b) 
$$\mathbf{F} = (y\cos x + \ln y)\mathbf{i} + \left(\frac{x}{y} + e^y\right)\mathbf{j}$$

Ex. 8. Find a potential function of the vector field and use the fundamental theorem for line integrals to evaluate

$$\int_{(\pi/2,\pi/2)}^{(\pi,\pi)} (\sin y + y \cos x) \, dx + (\sin x + x \cos y) \, dy =$$

Ex. 9. Apply Green's theorem to evaluate the following integral, where the simple closed curve C, with counter clockwise direction, is the boundary of the circle  $x^2 + y^2 = 1$ .

$$\oint_C (\sin x - x^2 y) \, dx + xy^2 \, dy =$$