MATH 251.007 NAME (print): ______________________

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SAMPLE TEST # 1

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Evaluate

(a) $$3\begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 11 & -1 \end{pmatrix} - 5\begin{pmatrix} 0 & 8 \\ 4 & -2 \\ 5 & 1 \end{pmatrix} =$$

(b) $$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 6 & 3 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 2 & 3 \end{pmatrix} =$$

(c) $$\begin{pmatrix} 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 7 \\ -1 \end{pmatrix} =$$

(d) $$\begin{pmatrix} 1 \\ 7 \\ -1 \end{pmatrix}\begin{pmatrix} 1 & -2 & 3 \end{pmatrix} =$$

Ex. 2. Solve each of the following systems of linear equations by representing as augmented matrix and transforming it to the row reduced echelon form. If the system inconsistent, give a reason for it explain the meaning of this fact in terms of solutions. If it is consistent, express its solution in the vertical vector *form* as $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix}$.

(a) $$\begin{cases} a + b - c = 0 \\ a - 4b + 2c = -1 \\ 2a - 3b + c = 1 \end{cases}$$

(b) $$\begin{cases} a + b - c = 0 \\ a - 4b + 2c = 1 \\ 2a - 3b + c = 1 \end{cases}$$

(c) $$\begin{cases} a + b - c = 0 \\ a + 2c = 1 \\ 2a - 3b + c = 1 \end{cases}$$

Ex. 3. Find the determinant of the matrix. Each time you expand the the matrix, you **must** expand it over a row or column that has the largest number of zeros. If necessary, use the row (or column) reduction method to create additional zeros.

$$\begin{pmatrix} -1 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 \end{pmatrix}$$

Ex. 4. Find the inverse matrix of

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix}$$

Ex. 5. Let A be as below. Show that it is its own inverse, that is, that $A^{-1} = A$.

$$A = \begin{pmatrix} 1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Ex. 6. Let $\mathbf{a} = \langle 0, 1, 2 \rangle$, $\mathbf{b} = \langle -1, 0, 7 \rangle$, and $\mathbf{c} = \langle 2, 3, -1 \rangle$. Evaluate: $2\mathbf{a} - \mathbf{b} + \mathbf{c}$, $|\mathbf{c}|$, and $(\mathbf{a} \cdot \mathbf{b})\ (\mathbf{b} \times \mathbf{c})$. (Do not confuse vectors with numbers. No partial credit for solutions with such errors.)