SAMPLE TEST # 3

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Find the general solution for the following differential equations:

(a) $y^{(4)} + 8y'' + 16y = 0$

(b) $y^{(4)} - 8y'' + 16y = 0$

(c) $y''' - 3y'' + 2y' = e^{-t}$

(d) $y''' - 3y'' + 2y' = t$

Ex. 2. Find the interval of convergence of the following series. Check the endpoints for extra credit. $\sum\limits_{n=1}^{\infty} \dfrac{(4x + 3)^n}{9n^2}$.

Ex. 3. Use power series with $x_0 = 1$ to solve $y'' + xy' + y = 0$. Find the recurrence formula and use it to find the first two non-zero terms in each of *two independent solutions.*

Ex. 4. Calculate the Laplace transform, $\mathcal{L}[f(t)]$, of the function $f(t) = 5t$. Show the details and use limits to evaluate any improper integrals.

Ex. 5. Use **Laplace transforms** to solve $y'' + 5y' - 6y = 3$, $y(0) = 0$, $y'(0) = 1$. Recall that $\mathcal{L}[e^{at}] = \frac{1}{s-a}$ for $s > a$. (For $a = 0$ this gives $\mathcal{L}[1] = \frac{1}{s}$.)