MATH 261.007
Instr. K. Ciesielski
Spring 2010

SAMPLE TEST # 1

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Each of the following differential equations is of one of the following form: linear, separable, homogenous, Bernouli, or exact. Solve each of these using appropriate method. (In the actual test, in some of such problems you will be asked only to *label the differential equation as one of these five kinds, including justification for the label,* without providing full solution.)

(a) $y' = \frac{e^x - e^{-x}}{3+4y}$, $y(0) = 1$

(b) $\frac{y}{x} + 6x + (\ln x - 2)\frac{dy}{dx} = 0$, $x > 0$

(c) $ty' - y = t^2 e^{-t}$, $t > 0$

(d) $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$

(e) $x^2 y' = y^3 - 2xy$, $x > 0$

(f) $\frac{dy}{dx} + y = \frac{1}{1+e^x}$

Ex. 2. Draw the direction field for the equation $\frac{dy}{dx} = 1 + y^2$. In your drawing show the places where the slopes are 1, 2, and 5.

Ex. 3. Without solving, determine the largest interval in which the initial value problem $(x^2 - x - 6)y' + y\cos x = e^x$, $y(2) = 0$, has a unique solution.

Ex. 4. Apply Euler's method to $y' = \frac{4-ty}{1+y^2}$, $y(0) = -2$, with step $h = 0.1$ to estimate $y(0.1)$.

Ex. 5. A tank with a capacity of 500 gal originally contains 200 gal of wather with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flaw out of the tank at the rate of 2 gal/min. Write down an initial value problem (ODE plus initial condition) giving the amount of salt in the tank at any time during the first hour. **Do not solve the equation.** Remember to give the initial condition.