

**SAMPLE TEST # 3**

Solve the exercises. **Show your work.**

**Ex. 1.** Solve the initial value problem:  $y' = e^{x-y}$ ,  $y(0) = 1$ .

**Ex. 2.** Find the arc length of the curve:  $y = x^2 - \frac{\ln x}{8}$ ,  $1 \leq x \leq 4$ .

**Ex. 3.** Find the area of the surface obtained by rotating the curve  $y = x^2$  from  $(1, 1)$  to  $(2, 4)$  about the  $y$ -axis.

**Ex. 4.** Find the centroid of the region bounded by the curves  $y = \cos x$ ,  $y = 0$ ,  $x = 0$ , and  $x = \pi/2$ .

**Ex. 5.** Eliminate the parameter from the equations  $x = \sin t$  and  $y = \sin^2 t$ . Sketch its graph.

**Ex. 6.** Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for  $x = t^2$  and  $y = t^3 - 3t$ .

**Ex. 7.** Find the surface area generated by rotating the curve  $x = e^t - t$ ,  $y = 4e^{t/2}$ ,  $0 \leq t \leq 1$ , about the  $x$ -axis.

**Ex. 8.** Change to polar coordinates. **Simplify your answer.**

$$x^2 + y^2 = 2x + 4.$$

**Ex. 9.** Change to Cartesian (rectangular) coordinates. **Simplify your answer.**

$$r = 2 \cos \theta - 3 \sin \theta.$$

**Ex. 10.** Find the slope of the tangent line to the curve:  $r = 1 + \sin \theta$  for  $\theta = \pi/3$ .

**Ex. 11.** Find the area of the region that lies inside the curve  $r = 3 \sin \theta$  and outside the curve  $r = 1 + \sin \theta$ .

**Ex. 12.** Find the equation of the parabola with focus  $(1, -1)$  and directrix  $y = 5$ .