MATH 251
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SAMPLE TEST # 4

Solve the following exercises. **Show your work.**

Ex. 1. Set up the integral formulas, **including the limits of the integrations,** for the following problems. *Do not evaluate the integrals!* Where appropriate, *use polar, cylindrical, or spherical coordinates.*

(a) The volume of the solid bounded by $z = x^2 + y^2$, $z = 0$, $x = 0$, $y = 0$, and $x + y = 1$.

(b) The mass of the plane lamina bounded by $y = x^2$ and $y = 2x + 3$, with the density $\delta(x, y) = x^2$.

(c) The mass of the solid T with the density $\delta(x, y, z) = x^2 + e^z$ bounded by the surfaces: $6x + 2y + z = 12$, $x = 0$, $y = 0$, and $z = 0$.

Ex. 2. Evaluate the integrals:

(a) $\displaystyle\int_0^1 \int_0^\pi \frac{1}{x+1} + \sin y \; dy \; dx =$

(b) $\displaystyle\int_{-2}^0 \int_0^y (x + 2y^2) \; dx \; dy =$

(c) $\displaystyle\int\int_R \frac{dy \; dx}{\sqrt{9 - x^2 - y^2}}$, where R is the *second quadrant* region bounded by $x^2 + y^2 = 4$.

Ex. 3. Find the mass of the solid bounded by the hemisphere $x^2 + y^2 + z^2 \leq R^2$, $z \geq 0$, with the density $\delta(x, y, z) = x^2 + y^2 + z^2$.

Ex. 4. Find the mass of the plane lamina bounded by $x = 0$ and $x = 9 - y^2$ with density $\delta(x, y) = x^2$.

Ex. 5. Evaluate $\displaystyle\int_C xy \, ds$, where C is the parametric curve for which $x = 3t$, $y = t^4$, and $0 \leq t \leq 1$.

Ex. 6. Evaluate the integral, where C is the graph of $y = x^3$ from $(-1, -1)$ to $(1, 1)$

$\displaystyle\int_C y^2 \, dx + x dy =$

Ex. 7. Determine if the following vector field is conservative. Find potential function for a field, if it is conservative.

(a) $\mathbf{F} = \left(x^3 + \frac{y}{x}\right) \mathbf{i} + (y^2 + \ln x)\mathbf{j}$

(b) $\mathbf{F} = (y \cos x + \ln y) \, \mathbf{i} + \left(\frac{x}{y} + e^y\right) \mathbf{j}$

Ex. 8. Find a potential function of the vector field and use the fundamental theorem for line integrals to evaluate

$$\int_{(\pi/2,\pi/2)}^{(\pi,\pi)} (\sin y + y\cos x)\,dx + (\sin x + x\cos y)\,dy =$$

Ex. 9. **Apply Green's theorem** to evaluate the following integral, where the simple closed curve C, with counter clockwise direction, is the boundary of the circle $x^2 + y^2 = 1$.

$$\oint_C (\sin x - x^2 y)\,dx + xy^2\,dy =$$