

**SAMPLE TEST # 2**

Solve the following exercises. **Show your work.**

**Ex. 1.** Find a vector equation of the line that passes through the point  $P(11, 13, -7)$  and is perpendicular to the plane with the equation:  $x - 2z = 17$ .

**Ex. 2.** Find: (a) the *unit* tangent vector to the curve  $\mathbf{r}(t) = \langle e^t, t, \cos \pi t \rangle$  at the point  $(1, 0, 1)$ , and (b) the vector equation of the line tangent to the same curve at the point  $(e, 1, -1)$ .

**Ex. 3.** Find the volume of the pyramid with the vertices:  $P(3, 2, -1)$ ,  $Q(-2, 5, 1)$ ,  $R(2, 1, 5)$ , and the origin  $O(0, 0, 0)$ . The volume of a pyramid is equal 1/6th of the volume of parallelepiped spanned by the same vectors.

**Ex. 4.** Find an equation of the plane passing through point  $(1, 11, -13)$  and parallel to the plane with equation  $2x - 17z + \pi = 0$ .

**Ex. 5.** Describe and sketch the graphs of the surfaces given by the following equations. Name each surface. Give specific informations, like center and radius in the case of a sphere.

(a)  $2x^2 + 2y^2 + 2z^2 = 7x + 9y + 11z$

(b)  $4y = x^2 + z^2$

(c)  $4y = z^2$

**Ex. 6.** Find the curvature  $\kappa(t)$  of the curve with position vector  $\mathbf{r}(t) = \mathbf{i} \cos t + \mathbf{j} \sin t + 2t \mathbf{k}$ .

**Ex. 7.** Let  $\mathbf{v}(t) = \mathbf{i}(t + e)^{-1} + \mathbf{k} t^3$  be a velocity of a particle. Find the acceleration vector  $\mathbf{a}(t)$  of the particle and its position vector  $\mathbf{r}(t)$ , where its initial position was  $\mathbf{r}(0) = 3\mathbf{i}$ .

**Ex. 8.** Find the arc length,  $s$ , of the curve with position vector  $\mathbf{r}(t) = 2e^t \mathbf{i} + 2t \mathbf{j} + e^{-t} \mathbf{k}$  from  $t = 0$  to  $t = 1$ .

**Ex. 9.** Sketch and fully describe the graph of a function  $f(x, y) = \sqrt{1 + x^2 + y^2}$ .

**Ex. 10.** Sketch and fully describe the domain of the following function, including the name of the surface representing the domain's boundary:  $f(x, y, z) = \ln(25 - 4x^2 - 9y^2 - z^2)$ .