MATH 251
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Fall 2017

SAMPLE FINAL TEST
(longer than the actual Final Test)

Solve the following exercises. **Show your work.**

Ex. 1. ST #1 Ex 3: Find the determinant of the matrix. Each time you expand the the matrix, you **must** expand it over a row or column that has the largest number of zeros. If necessary, use the row (or column) reduction method to create additional zeros.

$$A = \begin{bmatrix} -1 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 \end{bmatrix}$$

Ex. 2. ST #1 Ex 4: Find the inverse matrix of

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

Ex. 3. ST #1 Ex 6: Let $\mathbf{a} = \langle 0, 1, 2\rangle$, $\mathbf{b} = \langle -1, 0, 7\rangle$, and $\mathbf{c} = \langle 2, 3, -1\rangle$. Evaluate: $2\mathbf{a} - \mathbf{b} + \mathbf{c}$, $|\mathbf{c}|$, and $(\mathbf{a} \cdot \mathbf{b})\ (\mathbf{b} \times \mathbf{c})$. (Do not confuse vectors with numbers. No partial credit for solutions with such errors.)

Ex. 4. ST #2 Ex 1: Find a vector equation of the line that passes through the point $P(11, 13, -7)$ and is perpendicular to the plane with the equation: $x - 2z = 17$.

Ex. 5. ST #2 Ex 7: Let $\mathbf{v}(t) = \mathbf{i}(t + e)^{-1} + \mathbf{k}\ t^3$ be a velocity of a particle. Find the acceleration vector $\mathbf{a}(t)$ of the particle and its position vector $\mathbf{r}(t)$, where its initial position was $\mathbf{r}(0) = 3\mathbf{i}$.

Ex. 6. ST #2 Ex 10: Sketch and fully describe the domain of the following function, including the name of the surface representing the domain's boundary: $f(x, y, z) = \ln(25 - 4x^2 - 9y^2 - z^2)$.

Ex. 7. ST #3 Ex. 2: Compute the first order partial derivatives of $f(x, y, z) = ze^{x^2} \cos y$.

Ex. 8. ST #3 Ex. 3: Compute all second order partial derivatives of $g(s, t) = e^{5t} + t \sin(3s)$.

Ex. 9. ST #3 Ex. 4: Find an equation of the plane tangent to the surface $z = x^2 - 5y^3$ at the point $P(2, 1, -1)$.

Ex. 10. ST #3 Ex. 8: Find the point on the cone $z = \sqrt{x^2 + y^2}$ which is the closest to the point $(4, -8, 0)$.

Ex. 11. ST #3 Ex. 5: Find the absolute maximum and the absolute minimum of the function $f(x, y) = x^3 - xy$ on the region bounded below by parabola $y = x^2 - 1$ and above by line $y = 3$. You will get credit **only** if **all** critical points are found.

Ex. 12. ST #4 Ex. 1(a)&(c): Set up the integral formulas, **including the limits of the integrations,** for the following problems. *Do not evaluate the integrals!*

(a) The volume of the solid bounded by $z = x^2 + y^2$, $z = 0$, $x = 0$, $y = 0$, and $x + y = 1$.

(c) The mass of the solid T with the density $\delta(x, y, z) = x^2 + e^z$ bounded by the surfaces: $6x + 2y + z = 12$, $x = 0$, $y = 0$, and $z = 0$.

Ex. 13. ST #4 Ex. 2: Evaluate the integrals:

(a) $\displaystyle\int_0^1 \int_0^\pi \frac{1}{x+1} + \sin y \; dy \; dx =$

(b) $\displaystyle\int_{-2}^0 \int_0^y (x + 2y^2) \; dx \; dy =$

(c) $\displaystyle\int\int_R \frac{dy \; dx}{\sqrt{9 - x^2 - y^2}}$, where R is the *second quadrant* region bounded by $x^2 + y^2 = 4$.

Ex. 14. ST #4 Ex. 3: Find the mass of the solid bounded by the hemisphere $x^2+y^2+z^2 \leq R^2$, $z \geq 0$, with the density $\delta(x, y, z) = x^2 + y^2 + z^2$.

Ex. 15. ST #4 Ex. 4: Find the mass of the plane lamina bounded by $x = 0$ and $x = 4-y^2$ with density $\delta(x, y) = y^2$.

Ex. 16. ST #4 Ex. 6: Evaluate the integral, where C is the graph of $y = x^3$ from $(-1, -1)$ to $(1, 1)$

$$\int_C y^2 \; dx + x dy =$$

Ex. 17. ST #4 Ex. 8: Find a potential function of the vector field and use the fundamental theorem for line integrals to evaluate

$$\int_{(\pi/2, \pi/2)}^{(\pi, \pi)} (\sin y + y \cos x) \; dx + (\sin x + x \cos y) \; dy =$$

Ex. 18. ST #4 Ex. 9: Apply Green's theorem to evaluate the following integral, where the simple closed curve C, with counter clockwise direction, is the boundary of the circle $x^2 + y^2 = 1$.

$$\oint_C (\sin x - x^2 y) \; dx + xy^2 \; dy =$$