

SAMPLE TEST # 4

Solve the following exercises. **Show your work.**

**Ex. 1.** Set up the integral formulas, **including the limits of the integrations**, for the following problems. *Do not evaluate the integrals!* Where appropriate, use *polar, cylindrical, or spherical coordinates*.

- (a) The volume of the solid bounded by  $z = x^2 + y^2$ ,  $z = 0$ ,  $x = 0$ ,  $y = 0$ , and  $x + y = 1$ .
- (b) The mass of the plane lamina bounded by  $y = x^2$  and  $y = 2x + 3$ , with the density  $\delta(x, y) = x^2$ .
- (c) The mass of the solid  $T$  with the density  $\delta(x, y, z) = x^2 + e^z$  bounded by the surfaces:  $6x + 2y + z = 12$ ,  $x = 0$ ,  $y = 0$ , and  $z = 0$ .

**Ex. 2.** Evaluate the integrals:

- (a)  $\int_0^1 \int_0^\pi \frac{1}{x+1} + \sin y \, dy \, dx =$
- (b)  $\int_{-2}^0 \int_0^y (x + 2y^2) \, dx \, dy =$
- (c)  $\int \int_R \frac{dy \, dx}{\sqrt{9 - x^2 - y^2}}$ , where  $R$  is the *second quadrant* region bounded by  $x^2 + y^2 = 4$ .

**Ex. 3.** Find the mass of the solid bounded by the hemisphere  $x^2 + y^2 + z^2 \leq R^2$ ,  $z \geq 0$ , with the density  $\delta(x, y, z) = x^2 + y^2 + z^2$ .

**Ex. 4.** Find the mass of the plane lamina bounded by  $x = 0$  and  $x = 9 - y^2$  with density  $\delta(x, y) = x^2$ .

**Ex. 5.** Evaluate  $\int_C xy \, ds$ , where  $C$  is the parametric curve for which  $x = 3t$ ,  $y = t^4$ , and  $0 \leq t \leq 1$ .

**Ex. 6.** Evaluate the integral, where  $C$  is the graph of  $y = x^3$  from  $(-1, -1)$  to  $(1, 1)$

$$\int_C y^2 \, dx + x \, dy =$$

**Ex. 7.** Determine if the following vector field is conservative. Find potential function for a field, if it is conservative.

- (a)  $\mathbf{F} = \left(x^3 + \frac{y}{x}\right) \mathbf{i} + (y^2 + \ln x) \mathbf{j}$
- (b)  $\mathbf{F} = (y \cos x + \ln y) \mathbf{i} + \left(\frac{x}{y} + e^y\right) \mathbf{j}$

**Ex. 8.** Find a potential function of the vector field and use the fundamental theorem for line integrals to evaluate

$$\int_{(\pi/2, \pi/2)}^{(\pi, \pi)} (\sin y + y \cos x) dx + (\sin x + x \cos y) dy =$$

**Ex. 9. Apply Green's theorem** to evaluate the following integral, where the simple closed curve  $C$ , with counter clockwise direction, is the boundary of the circle  $x^2 + y^2 = 1$ .

$$\oint_C (\sin x - x^2 y) dx + xy^2 dy =$$