MATH 251
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SAMPLE TEST # 3

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Show that the following limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^4 + 7y^4}$$

Ex. 2. Compute the first order partial derivatives of $f(x, y, z) = ze^{x^2} \cos y$.

Ex. 3. Compute all second order partial derivatives of $g(s, t) = e^{5t} + t \sin(3s)$.

Ex. 4. Find an equation of the plane tangent to the surface $z = x^2 - 5y^3$ at the point $P(2, 1, -1)$.

Ex. 5. Find the absolute maximum and the absolute minimum of the function $f(x, y) = x^3 - xy$ on the region bounded below by parabola $y = x^2 - 1$ and above by line $y = 3$. You will get credit **only** if **all** critical points are found.

Ex. 6. Find the volume of the solid bounded above by the surface $z = 28xy$, bounded below by xy-plane, and which is above the region bounded by $y = x^6$ and $y = x$.

Ex. 7. Evaluate $\int_0^1 \int_0^x 4e^{x^2}\, dy\, dx$

Ex. 8. Find the point on the cone $z = \sqrt{x^2 + y^2}$ which is the closest to the point $(4, -8, 0)$.

Ex. 9. Find the directional derivative of $f(x, y) = 10e^y \sin x$ at the point $P(\pi/4, 0)$ in the direction of the vector $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$.

Ex. 10. Find the gradient of $g(x, y, z) = x^2 + e^{yz} + \cos(x + 2y)$.