MATH 251

NAME (print):

Instr. K. Ciesielski Fall 2014

## SAMPLE TEST # 1

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Evaluate

(a) 
$$3\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 11 & -1 \end{bmatrix} - 5\begin{bmatrix} 0 & 8 \\ 4 & -2 \\ 5 & 1 \end{bmatrix} =$$

(b) 
$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 6 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 2 & 3 \end{bmatrix} =$$

(c) 
$$\begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix} =$$

(d) 
$$\begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \end{bmatrix} =$$

Ex. 2. Solve each of the following systems of linear equations by representing as augmented matrix and transforming it to the row reduced echelon form. If the system inconsistent, give a reason for it explain the meaning of this fact in terms of solutions. If it is consistent, express

its solution in the vertical vector form as  $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$  or  $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} + y \begin{bmatrix} 0 \\ 5 \\ 11 \end{bmatrix}$ .

(a) 
$$\begin{cases} a + b - c = 0 \\ a - 4b + 2c = -1 \\ 2a - 3b + c = 1 \end{cases}$$

(b) 
$$\begin{cases} a + b - c = 0 \\ a - 4b + 2c = 1 \\ 2a - 3b + c = 1 \end{cases}$$

(c) 
$$\begin{cases} a + b - c = 0 \\ a + 2c = 1 \\ 2a - 3b + c = 1 \end{cases}$$

Ex. 3. Find the determinant of the matrix. Each time you expand the matrix, you must expand it over a row or column that has the largest number of zeros. If necessary, use the row (or column) reduction method to create additional zeros.

$$A = \left[ \begin{array}{rrrr} -1 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 \end{array} \right]$$

Ex. 4. Find the inverse matrix of

$$A = \left[ \begin{array}{rrr} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & 1 & -1 \end{array} \right]$$

**Ex. 5.** Let A be as below. Show that it is its own inverse, that is, that  $A^{-1} = A$ .

$$A = \begin{bmatrix} 1/2 & \sqrt{3}/2 & 0\\ \sqrt{3}/2 & -1/2 & 0\\ 0 & 0 & -1 \end{bmatrix}$$

**Ex. 6.** Let  $\mathbf{a} = \langle 0, 1, 2 \rangle$ ,  $\mathbf{b} = \langle -1, 0, 7 \rangle$ , and  $\mathbf{c} = \langle 2, 3, -1 \rangle$ . Evaluate:  $2\mathbf{a} - \mathbf{b} + \mathbf{c}$ ,  $|\mathbf{c}|$ , and  $(\mathbf{a} \cdot \mathbf{b})$  ( $\mathbf{b} \times \mathbf{c}$ ). (Do not confuse vectors with numbers. No partial credit for solutions with such errors.)