

MATH 251.007
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SAMPLE TEST # 3

Solve the following exercises. **Show your work.**

Ex. 1. Show that the following limit does not exist

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{3x^2 + 4y^2} =$$

Ex. 2. Compute the first order partial derivatives of $h(x, y, z) = e^{2x+3z} \sin x \tan y$.

Ex. 3. Compute the second order partial derivatives of $g(u, v) = \ln(u + 2v) - \sin u \cos v$.

Ex. 4. Find an equation of the plane tangent to the surface $z = \ln x - \sin y$ at the point $P(1, \pi/2, -1)$.

Ex. 5. Find the absolute maximum and the absolute minimum of the function $f(x, y) = 4x^2 + 2xy + y^2$ on the region bounded below by the parabola $y = x^2$ and above by the line $y = 9$.

Ex. 6. Find the gradient of $g(x, y, z) = x^2 + e^{yz} + \cos(x + 2y)$.

Ex. 7. Find the first octant point on the surface $xyz = 8$ which is the closest to the point $P(0, 0, 0)$. (The first octant is the set of points (x, y, z) with $x \geq 0$, $y \geq 0$, and $z \geq 0$.)

Ex. 8. Find the directional derivative of $f(x, y) = \sin x \cos y$ at the point $P(\pi/3, -2\pi/3)$ in the direction of the vector $\mathbf{v} = \langle 4, -3 \rangle$.

Ex. 9. Find the volume of the solid bounded by the surfaces: $z = x^2 + 3y^2$, $x = 0$, $y = 1$, $y = x$, and $z = 0$.

Ex. 10. Evaluate the integrals:

(a) $\int_{-1}^2 \int_{-y}^0 (x + 2y^2) dx dy =$

(b) $\int_0^1 \int_y^1 e^{-x^2} dx dy =$