

**SAMPLE TEST # 1**

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

**Ex. 1.** Evaluate

$$(a) \quad 3 \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 11 & -1 \end{pmatrix} - 5 \begin{pmatrix} 0 & 8 \\ 4 & -2 \\ 5 & 1 \end{pmatrix} =$$

$$(b) \quad \begin{pmatrix} 1 & 2 & 1 \\ -1 & 6 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 2 & 3 \end{pmatrix} =$$

$$(c) \quad \begin{pmatrix} 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 7 \\ -1 \end{pmatrix} =$$

$$(d) \quad \begin{pmatrix} 1 \\ 7 \\ -1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \end{pmatrix} =$$

**Ex. 2.** Solve each of the following systems of linear equations by representing as augmented matrix and transforming it to the row reduced echelon form. If the system inconsistent, give a reason for it explain the meaning of this fact in terms of solutions. If it is consistent, express

its solution in the vertical vector *form* as  $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$  or  $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + y \begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix}$ .

$$(a) \quad \begin{cases} a + b - c = 0 \\ a - 4b + 2c = -1 \\ 2a - 3b + c = 1 \end{cases}$$

$$(b) \quad \begin{cases} a + b - c = 0 \\ a - 4b + 2c = 1 \\ 2a - 3b + c = 1 \end{cases}$$

$$(c) \quad \begin{cases} a + b - c = 0 \\ a + 2c = 1 \\ 2a - 3b + c = 1 \end{cases}$$

**Ex. 3.** Find the determinant of the matrix. Each time you expand the the matrix, you **must** expand it over a row or column that has the largest number of zeros. If necessary, use the row (or column) reduction method to create additional zeros.

$$\begin{pmatrix} -1 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 \end{pmatrix}$$

**Ex. 4.** Find the inverse matrix of

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix}$$

**Ex. 5.** Let  $A$  be as below. Show that it is its own inverse, that is, that  $A^{-1} = A$ .

$$A = \begin{pmatrix} 1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

**Ex. 6.** Let  $\mathbf{a} = \langle 0, 1, 2 \rangle$ ,  $\mathbf{b} = \langle -1, 0, 7 \rangle$ , and  $\mathbf{c} = \langle 2, 3, -1 \rangle$ . Evaluate:  $2\mathbf{a} - \mathbf{b} + \mathbf{c}$ ,  $|\mathbf{c}|$ , and  $(\mathbf{a} \cdot \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c})$ . (Do not confuse vectors with numbers. No partial credit for solutions with such errors.)