MATH 261.005
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Bernouli Equations

Format: $y' + p(t)y = g(t)y^n$.
Solution:

(a) For $n = 0$ and $n = 1$ this is linear equation. For other n use substitution $v = y^{1-n}$.

(b) Since $y = v^{\frac{1}{1-n}}$, taking the derivative we get $y' = \frac{1}{1-n} v^{\frac{1}{1-n}-1} v' = \frac{1}{1-n} v^{\frac{n}{1-n}} v'$.

(c) Substituting this to $y' + p(t)y = g(t)y^n$ we get
$\frac{1}{1-n} v^{\frac{n}{1-n}} v' + p(t) v^{\frac{1}{1-n}} = g(t) \left(v^{\frac{1}{1-n}}\right)^n$.

(d) Multiplying this by the reciprocal of $\frac{1}{1-n} v^{\frac{n}{1-n}}$, that is by $(1-n)v^{-\frac{n}{1-n}}$, we get
$v' + (1-n)p(t) v^{\frac{1}{1-n}} v^{-\frac{n}{1-n}} = (1-n) g(t) v^{\frac{n}{1-n}} v^{-\frac{n}{1-n}}$, that is,
$v' + (1-n)p(t)v = (1-n)g(t)$,
since $\frac{1}{1-n} - \frac{n}{1-n} = 1$. *Algebra always leads to the linear equation!*

(e) Next, solve (for v) the linear equation $v' + (1-n)p(t)v = (1-n)g(t)$.

(f) Using again equation $y = v^{\frac{1}{1-n}}$, find the formula for y from the formula for v found in (e).

Types of First Order Differential Equations we studied:

linear: $y' + p(t)y = g(t)$

separable: $M(x) = N(y)\,\frac{dy}{dx}$, leading to $\int M(x)\,dx = \int N(y)\,dy$

homogenous: $\frac{dy}{dx} = h(y/x)$; use substitution $v = y/x$ and fact that $\frac{dy}{dx} = v + x\frac{dv}{dx}$;
the resulted equation is separable (in terms of v and x)

Bernouli: $y' + p(t)y = g(t)y^n$; appropriate substitution (see above) leads to linear ODE

exact: $M(x,y) + N(x,y)y' = 0$ (or $M(x,y)\,dx + N(x,y)\,dy = 0$), when $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$;
then the solution is of the form $\Psi(x,y) = c$, where $\frac{\partial \Psi}{\partial x} = M$ and $\frac{\partial \Psi}{\partial y} = N$.