

SAMPLE TEST # 4

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Transform the following system of equations into a single second order equation in terms of x_1 . Then give the initial condition for the resulted equation that corresponds to the given initial conditions. Do not solve.

$$x_1' = -0.5x_1 + 2x_2; \quad x_2' = -2x_1 - 0.5x_2; \quad x_1(0) = -2, \quad x_2(0) = 2.$$

Ex. 2. Use eigenvalues and eigenvectors to find the general solution of the given systems of differential equations. The solution must be expressed in terms of real-valued functions.

$$(a) \quad \mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \mathbf{x}$$

$$(b) \quad \mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}$$

$$(c) \quad \mathbf{x}' = \begin{pmatrix} 6 & -3 \\ 3 & 0 \end{pmatrix} \mathbf{x}$$

Ex. 3. Solve the following boundary value problem or show that it does not have a solution.
 $y'' + 4y = 0$, $y(0) = 0$, $y(\pi) = 0$.

Ex. 4. Determine whether the method of separation of variables can be used to replace the partial differential equation $u_{xx} + u_{xt} + u_t = 0$ by a pair of ordinary differential equations. If so, find the ordinary differential equations. Do not solve them.

Ex. 5. Solve the heat equation: $u_t = 9u_{xx}$, $u(0, t) = u(2, t) = 0$, $u(x, 0) = 13$ for $0 < x < 2$.