

**SAMPLE TEST # 3**

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

**Ex. 1.** Find the general solution for the following differential equations:

(a)  $y^{(4)} + 8y'' + 16y = 0$

(b)  $y^{(4)} - 8y'' + 16y = 0$

(c)  $y''' - 3y'' + 2y' = e^{-t}$

(d)  $y''' - 3y'' + 2y' = t$

**Ex. 2.** Find the interval of convergence of the following series. Check the endpoints for extra credit.  $\sum_{n=1}^{\infty} \frac{(4x + 3)^n}{9n^2}$ .

**Ex. 3.** Use power series with  $x_0 = 1$  to solve  $y'' + xy' + y = 0$ . Find the recurrence formula and use it to find the first two non-zero terms in each of *two independent solutions*.

**Ex. 4.** Calculate the Laplace transform,  $\mathcal{L}[f(t)]$ , of the function  $f(t) = 5t^2$ . Show the details and use limits to evaluate any improper integrals.

**Ex. 5.** Use **Laplace transforms** to solve  $y'' + 5y' - 6y = 3$ ,  $y(0) = 0$ ,  $y'(0) = 1$ . Recall that  $\mathcal{L}[e^{at}] = \frac{1}{s-a}$  for  $s > a$ . (For  $a = 0$  this gives  $\mathcal{L}[1] = \frac{1}{s}$ .)