

MATH 251.007
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SAMPLE TEST # 4

Solve the following exercises. **Show your work.**

Ex. 1. Set up the integral formulas, **including the limits of the integrations**, for the following problems. *Do not evaluate the integrals!*

- (a) The volume of the solid bounded by $z = x^2 + y^2$, $z = 0$, $x = 0$, $y = 0$, and $x + y = 1$.
- (b) The mass of the plane lamina bounded by $y = x^2$ and $y = 2x + 3$, with the density $\delta(x, y) = x^2$.
- (c) The mass of the solid T with the density $\delta(x, y, z) = x^2 + e^z$ bounded by the surfaces: $6x + 2y + z = 12$, $x = 0$, $y = 0$, and $z = 0$.

Ex. 2. Evaluate the integrals:

- (a) $\int_0^1 \int_0^\pi \frac{1}{x+1} + \sin y \, dy \, dx =$
- (b) $\int_{-2}^0 \int_0^y (x + 2y^2) \, dx \, dy =$
- (c) $\int \int_R \frac{dy \, dx}{\sqrt{9 - x^2 - y^2}}$, where R is the *second quadrant* region bounded by $x^2 + y^2 = 4$.

Ex. 3. Find the mass of the solid bounded by the hemisphere $x^2 + y^2 + z^2 \leq R^2$, $z \geq 0$, with the density $\delta(x, y, z) = x^2 + y^2 + z^2$.

Ex. 4. Find the mass of the plane lamina bounded by $x = 0$ and $x = 9 - y^2$ with density $\delta(x, y) = x^2$.

Ex. 5. Evaluate $\int_C xy \, ds$, where C is the parametric curve for which $x = 3t$, $y = t^4$, and $0 \leq t \leq 1$.

Ex. 6. Evaluate the integral, where C is the graph of $y = x^3$ from $(-1, -1)$ to $(1, 1)$.

$$\int_C y^2 \, dx + x \, dy =$$

Ex. 7. Determine if the following vector field is conservative. Find potential function for a field, if it is conservative.

(a) $\mathbf{F} = \left(x^3 + \frac{y}{x}\right) \mathbf{i} + (y^2 + \ln x) \mathbf{j}$

(b) $\mathbf{F} = (y \cos x + \ln y) \mathbf{i} + \left(\frac{x}{y} + e^y\right) \mathbf{j}$

Ex. 8. Find a potential function of the vector field and use the fundamental theorem for line integrals to evaluate

$$\int_{(\pi/2, \pi/2)}^{(\pi, \pi)} (\sin y + y \cos x) \, dx + (\sin x + x \cos y) \, dy =$$

Ex. 9. Apply Green's theorem to evaluate the following integral, where the simple closed curve C is the boundary of the circle $x^2 + y^2 = 1$.

$$\oint_C (\sin x - x^2 y) \, dx + xy^2 \, dy =$$