SAMPLE TEST # 3

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Find the general solution for the following differential equations:

(a) $y^{(8)} - 18y^{(4)} + 81y = 0$

(b) $y^{(8)} - 81y = 0$

(c) $y^{(4)} - 4y'' = t^2 + e^t$

Ex. 2. Find the interval of convergence of the following series. Check the endpoints.
$\sum_{n=1}^{\infty} \frac{(-1)^n (2x+3)^n}{7n}$.

Ex. 3. Use power series with $x_0 = 1$ to solve $y'' - xy' - y = 0$. Find the recurrence formula and use it to find the first two non-zero terms in each of two independent solutions.

Ex. 4. Calculate the Laplace transform, $\mathcal{L}[f(t)]$, of the function $f(t) = 3t^2$. Show the details and use limits to evaluate any improper integrals.

Ex. 5. Use Laplace transforms to solve $y'' + 3y' + 2y = 1$, $y(0) = 1$, $y'(0) = 0$. Recall that $\mathcal{L}[e^{at}] = \frac{1}{s-a}$ for $s > a$.