MATH 261.005 NAME (print): ______________________

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SAMPLE FINAL TEST

This is en excerpt from the previous Sample Tests. The actual Final Test will be considerably shorter.

Test #1 material

Ex. 1. Each of the following differential equations is of one of the following form: linear, separable, homogenous, Bernouli, or exact. Solve each of these using appropriate method.

(a) $y' = \frac{e^{-x}+e^x}{3+4y}$, $y(0) = 1$

(b) $\frac{y}{x} + 6x + (\ln x - 2)\frac{dy}{dx} = 0$, $x > 0$

(c) $ty' - y = t^2 e^{-t}$, $t > 0$

(d) $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$

(f) $\frac{dy}{dx} + y = \frac{1}{1+e^x}$

Ex. 2. Without solving, determine the largest interval in which the initial value problem $(x^2 - x - 6)y' + y\cos x = e^x$, $y(2) = 0$, has a unique solution.

Ex. 3. A tank with a capacity of 500 gal originally contains 200 gal of wather with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flaw out of the tank at the rate of 2 gal/min. Write down an initial value problem (ODE plus initial condition) giving the amount of salt in the tank at any time during the first hour. **Do not solve the equation.** Remember to give the initial condition.

Test #2 material

Ex. 4. Find the general solution for each of the following differential equations:

(a) $y'' + 10y' + 25 = 0$

(b) $y'' + 10y' + 25y = 0$

(c) $y'' + 10y' + 29y = 0$

(d) $y'' + 10y' + 24y = 0$

Ex. 5. Solve the initial value problem $y'' + y' - 2y = 2t$, $y(0) = 0$, $y'(0) = 1$.

Ex. 6. Find a particular solution of the equation $y'' + 3y = 3\sin 2t$.

Ex. 7. Given that $y_1(x) = e^x$ is a solution of the ODE $(x-1)y'' - xy' + y = 0$, $x > 0$, use the method of reduction of order to find a second independent solution of this equation.

Ex. 8. Use **the variation of parameters method** to find a particular solution of the equation $y'' + 4y' + 4y = t^{-2}e^{-2t}$, $t > 0$. (No credit for the solution found by another method.)

Test #3 material

Ex. 9. Find the general solution for the following differential equations:

(a) $y^{(8)} - 18y^{(4)} + 81y = 0$

(b) $y^{(4)} - 4y'' = t^2 + e^t$

Ex. 10. Use power series with $x_0 = 1$ to solve $y'' - xy' - y = 0$. Find the recurrence formula and use it to find the first two non-zero terms in each of two independent solutions.

Ex. 11. Use Laplace transforms to solve $y'' + 3y' + 2y = 1$, $y(0) = 1$, $y'(0) = 0$. Recall that $\mathcal{L}[e^{at}] = \frac{1}{s-a}$ for $s > a$.

Test #4 material

Ex. 12. Use eigenvalues and eigenvectors to find the general solution of the given systems of differential equations. The solution must be expressed in terms of real-valued functions.

(a) $\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \mathbf{x}$

(b) $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{pmatrix} 6 & -3 \\ 3 & 0 \end{pmatrix} \mathbf{x}$

Ex. 13. Solve the following boundary value problem or show that it does not have a solution. $y'' + 4y = 0$, $y(0) = 0$, $y(\pi) = 0$.

Ex. 14. Determine whether the method of separation of variables can be used to replace the partial differential equation $u_{xx} + u_{xt} + u_t = 0$ by a pair of ordinary differential equations. If so, find the ordinary differential equations. Do not solve them.

Ex. 15. Solve the heat equation: $u_t = 9u_{xx}$, $u(0,t) = u(2,t) = 0$, $u(x,0) = 13$ for $0 < x < 2$.