MATH 251
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SAMPLE TEST # 2

Solve the following exercises. **Show your work.**

Ex. 1. Find the parametric equations of the line that passes through the point $P(11, 13, -7)$ and is perpendicular to the plane with the equation: $x - 2z = 17$.

Ex. 2. Find the unit normal vector to the curve $\mathbf{r}(t) = \langle e^t, t, \cos \pi t \rangle$ at the point $(1, 0, 1)$.

Ex. 3. Find the volume of the pyramid with the vertices: $P(3, 2, -1)$, $Q(-2, 5, 1)$, $R(2, 1, 5)$, and the origin $O(0, 0, 0)$. The volume of a pyramid id equal 1/6th of the volume of parallelepiped spanned by the same vectors.

Ex. 4. Find an equation of the plane passing through point $(1, 11, -13)$ and parallel to the plane with equation $2x - 17z + \pi = 0$.

Ex. 5. Describe and sketch the graphs of the surfaces given by the following equations. Name each surface. Give specific informations, like center and radius in the case of a sphere.

(a) $2x^2 + 2y^2 + 2z^2 = 7x + 9y + 11z$.

(b) $4y = x^2 + z^2$

(c) $4y = z^2$

Ex. 6. Find the curvature κ of the curve with position vector $\mathbf{r}(t) = \mathbf{i} \cos t + \mathbf{j} \sin t + 2t\, \mathbf{k}$.

Ex. 7. Let $\mathbf{v}(t) = \mathbf{i}(t + 1)^{-1} + \mathbf{k}t^3$ be a velocity of a particle. Find the acceleration vector $\mathbf{a}(t)$ of the particle and its position vector $\mathbf{r}(t)$, where its initial position was $\mathbf{r}_0 = 3\mathbf{i}$.

Ex. 8. Find the the arc length, s, of the curve with position vector $\mathbf{r}(t) = 2e^t\, \mathbf{i} + 2t\, \mathbf{j} + e^{-t}\, \mathbf{k}$ from $t = 0$ to $t = 1$.

Ex. 9. Sketch and fully describe the graph of a function $f(x, y) = \sqrt{1 + x^2 + y^2}$.