MATH 251.007
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SAMPLE FINAL TEST

Solve the following exercises. **Show your work.**

Ex. 1. Write the parametric equations of the line that passes through the point $P(11, 13, -7)$ and is perpendicular to the plane with the equation: $x - 2z = 17$.

Ex. 2. Let $\mathbf{a} = \langle 0, 1, 2\rangle$, $\mathbf{b} = \langle -1, 0, 7\rangle$, and $\mathbf{c} = \langle 2, 3, -1\rangle$. Evaluate $(\mathbf{a} \cdot \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c})$.

Ex. 3. Let $\mathbf{v}(t) = \mathbf{i}(t + 1)^{-1} + \mathbf{k}t^3$ be a velocity of a particle. Find the acceleration vector $\mathbf{a}(t)$ of the particle and its position vector $\mathbf{r}(t)$, where its initial position was $\mathbf{r}_0 = 3\mathbf{i}$.

Ex. 4. Describe and sketch the graph of the equation: $4z^2 = x^2 + y^2$.

Ex. 5. Compute the first order partial derivatives of $h(x, y, z) = e^{2x+3z} \sin x \tan y$.

Ex. 6. Compute the second order partial derivatives of $g(u, v) = \ln(u + 2v) - \sin u \cos v$.

Ex. 7. Find an equation of the plane tangent to the surface $z = \ln x - \sin y$ at the point $P(1, \pi/2, -1)$.

Ex. 8. Find the absolute maximum and the absolute minimum of the function $f(x, y) = 4x^2 + 2xy + y^2$ on the region bounded below by the parabola $y = x^2$ and above by the line $y = 9$.

Ex. 9. Evaluate the integrals:

(a) $\displaystyle\int_0^1 \int_0^\pi \frac{1}{x+1} + \sin y \, dy \, dx =$

(b) $\displaystyle\int_{-1}^2 \int_{-y}^{y+2} (x + 2y^2) \, dx \, dy =$

(c) $\displaystyle\int\int_R \frac{dy \, dx}{\sqrt{9 - x^2 - y^2}}$, where R is the second quadrant region bounded by $x^2 + y^2 = 4$.

Ex. 10. Find the mass of the solid bounded by the hemisphere $x^2 + y^2 + z^2 \leq R^2$, $z \geq 0$, with the density $\delta(x, y, z) = x^2 + y^2 + z^2$.

Ex. 11. Find the mass of the plane lamina bounded by $x = 0$ and $x = 9 - y^2$ with density $\delta(x, y) = x^2$.

Ex. 12. Set up the integral formulas, **including the limits of the integrations,** for the following problems. *Do not evaluate the integrals!*

(a) The mass of the solid T with the density $\delta(x, y, z) = x^2 + e^z$ bounded by the surfaces: $6x + 2y + z = 12$, $x = 0$, $y = 0$, and $z = 0$.

(b) The volume of the solid bounded by $z = x^2 + y^2$, $z = 0$, $x = 0$, $y = 0$, and $x + y = 1$.

Ex. 13. Evaluate $\int_C xy\,ds$, where C is the parametric curve for which $x = 3t$, $y = t^4$, and $0 \le t \le 1$.

Ex. 14. Evaluate the integral, where C is the graph of $y = x^3$ from $(-1, -1)$ to $(1, 1)$.

$$\int_C y^2\,dx + x dy =$$

Ex. 15. Evaluate the integral

$$\int_{(\pi/2,\pi/2)}^{(\pi,\pi)} (\sin y + y\cos x)\,dx + (\sin x + x\cos y)\,dy =$$

Ex. 16. Apply Green's theorem to evaluate the following integral, where the simple closed curve C is the boundary of the circle $x^2 + y^2 = 1$.

$$\oint_C (\sin x - x^2 y)\,dx + xy^2\,dy =$$

Ex. 17. Find the inverse matrix of

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Ex. 18. Find the determinant of the matrix

$$\begin{bmatrix} 1 & 2 & -11 & 1 \\ 0 & 0 & 2 & 0 \\ 7 & 1 & 0 & 2 \\ -2 & 0 & 9 & 0 \end{bmatrix}$$