Graph-based Segmentation with Local Band Constraints

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Outline

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Introduction

- Image Segmentation - Recognition and delineation
- Graph Based Segmentation
- Energy Optimization
- High-level Constraints

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Graph Based Image Segmentation

- Graph partition problem.
- Subject to hard constraints (seed markers).

Energy Optimizers

In this work, we are interested in an energy optimality criterion which is defined by a graph-cut measure. Two important classes of energy optimization on the GGC framework are:

- Max-Min (e.g.: OIFT, ORFC)
- Min-Sum (e.g.: Graph-Cuts)

Our algorithms are based on OIFT, ensuring that the segmentation with Max-Min energy is found. The resulting segmentation gives a global optimum solution by maximizing the following graph-cut measure, subject to the seed constraints.

$$\varepsilon_{\min}(L) \quad = \quad \min\{\omega(s,t) \colon (s,t) \in \mathcal{A} \ \& \ L(s) > L(t)\} \qquad (1)$$

High-Level Constraints

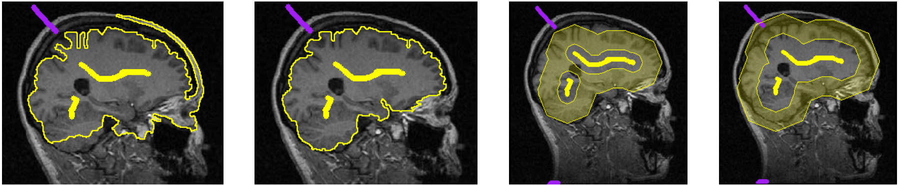
High Level constraints intended to regularize object borders are common in literature, some examples are:

- Geodesic Star Constraint (GSC) [Gulshan et.al., 2010]
  Directs the target object to have a star convex shape.
- Hedgehog Shape Prior (HSP) [Isack et.al., 2016]
  Uses the gradient of a distance transform from the seeds (i.e. a vector field) to avoid abrupt angle variations on the border.
- Boundary Band Constraint (BB) [Braz & Miranda, 2014]
  Prevents the generated segmentation to be irregular in relation to the level sets of a given reference cost map,by setting a maximum allowed variation between any two points of the boundary.

Boundary Band Constraint (BB) - Definition

For $\Delta > 0$ and a cost map $C : \mathcal{I} \rightarrow [0, \infty)$, a pixel $t \in \mathcal{O}$ is BB_Δ provided $C(t) < C(s) + \Delta$ for all $s \in \text{bd}(\mathcal{O})$.

An object $\mathcal{O}$ is BB_Δ provided every $t \in \mathcal{O}$ is BB_Δ.



(a) (b) (c) (d)

Figure: (a-b) Segmentation results by OIFT without and with the BB constraint, respectively. (c-d) The BB fixed size band evolves from the seeds, adapting to the image contents.

Boundary Band Constraint (BB) - Drawbacks

- Local changes can generate constraint violations in any other part of its border.
- Can result in greater sensitivity to the initialization of the cost map C and to the positioning of object seeds.

In order to address these issues, we can limit locally the constraint checks, leading to the Local Boundary Band Constraint.

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Local Boundary Band Constraint (LBB) - Definition

For $\Delta, R > 0$ and a cost map $C\colon \mathcal{I} \to [0, \infty)$, a pixel $t \in \mathcal{O}$ is LBB_Δ^R provided $C(t) < C(s) + \Delta$ for all $s \in \mathrm{bd}(\mathcal{O})$ such that $\|s - t\| \leq R$.

An object $\mathcal{O}$ is LBB_Δ^R provided every $t \in \mathcal{O}$ is LBB_Δ^R.

- Consistency checks are limited locally.
- $\mathcal{O}$ is $\mathrm{BB}_\Delta \implies \mathcal{O}$ is LBB_Δ^R.
- BB_Δ is the limit case of LBB_Δ^R, when $R \to \infty$.

Local Boundary Band Constraint (LBB) - Issues

- Computationally expensive.
- Analysis of the dynamic set of $\mathrm{bd}(\mathcal{O})$ inside the radius R at runtime.

In order to address this we need an approximate alternative definition.

Local Band Constraint (LB) - Definition

For $\Delta, R > 0$ and a cost map $C\colon \mathcal{I} \to [0, \infty)$, a pixel $t \in \mathcal{O}$ is LB_{Δ}^{R} provided $C(t) < C(s) + \Delta$ for all $s \in \mathcal{I} \setminus \mathcal{O}$ such that $\|s - t\| \leq R$.

An object $\mathcal{O}$ is LB_{Δ}^{R} provided every $t \in \mathcal{O}$ is LB_{Δ}^{R}.

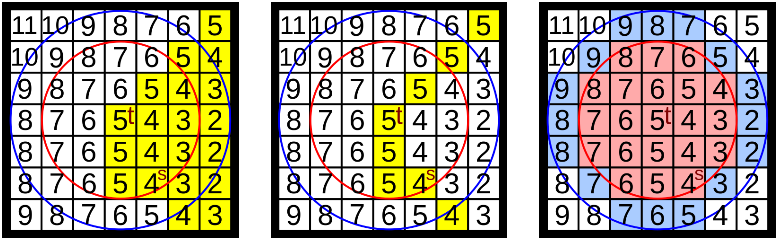
- Computationally feasible to implement.
- Graph preprocessing.

Relation between LBB and LB

Proposition

Let $r = \max_{(s,t)\in\mathcal{A}} \|s - t\|$ *and* $\delta = \max_{(s,t)\in\mathcal{A}} |C(t) - C(s)|$. *If* $\Delta, R > 0$ *and* $\mathcal{O}$ *is* $\mathrm{LB}_{\Delta}^{R+r}$, *then* $\mathcal{O}$ *is* $\mathrm{LBB}_{\Delta+\delta}^{R}$.

Since usually δ and r are small, so should be the difference between objects that are LB_{Δ}^{R}, $\mathrm{LB}_{\Delta}^{R+r}$, $\mathrm{LBB}_{\Delta+\delta}^{R}$, or $\mathrm{LBB}_{\Delta}^{R}$.



(a) $\mathcal{O}$ (b) $\mathrm{bd}(\mathcal{O})$ (c) R and $R + r$

Figure: t is $\mathrm{LB}_{\Delta}^{R+r}$ and t is $\mathrm{LBB}_{\Delta+\delta}^{R}$ for $R = 2.5$, $r = 1.0$, $\Delta = 1$ and $\delta = 1$.

LB Optimallity

Theorem

Let $G = (\mathcal{I}, \mathcal{A}, w)$ be a symmetric edge weighted image digraph with $w\colon \mathcal{A} \to \mathbb{R}$. Let L be a segmentation returned by Algorithm LB-OIFT applied to G, non-empty disjoint seed sets $\mathcal{S}_1$ and $\mathcal{S}_0$, cost map $C\colon \mathcal{I} \to [0, \infty)$, and parameters $R > 0$ and $\Delta > 0$. Assume that $\mathcal{S}_1$ and $\mathcal{S}_0$ are LB^R_Δ-consistent, that is, that there exists a labeling satisfying seeds and LB^R_Δ constraints.
Then L satisfies seeds and LB^R_Δ constraints and maximizes the energy $\varepsilon_{\min}$, given by (1) w.r.t. G, among all segmentations satisfying these constraints.

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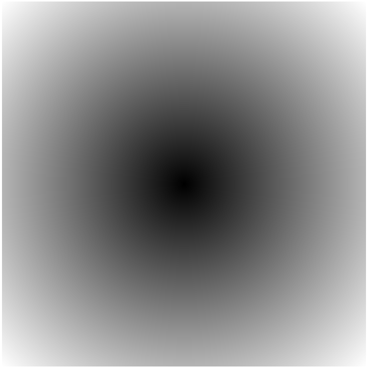
Experiments

We compared LB with shape constraints commonly employed in graph-based segmentation: GSC, HSP and BB. We opted to compare them using Max-Min optimizers, since BB is not yet supported by Min-Sum optimizers

We also tested their robustness in relation to different image resolutions by quantitative experiments, to segment archaeological fragments in seven different resolutions with the geodesic cost.

Shape Template



(a) Circle template

(b) Square template

Figure: Shape templates used.

Circle Template

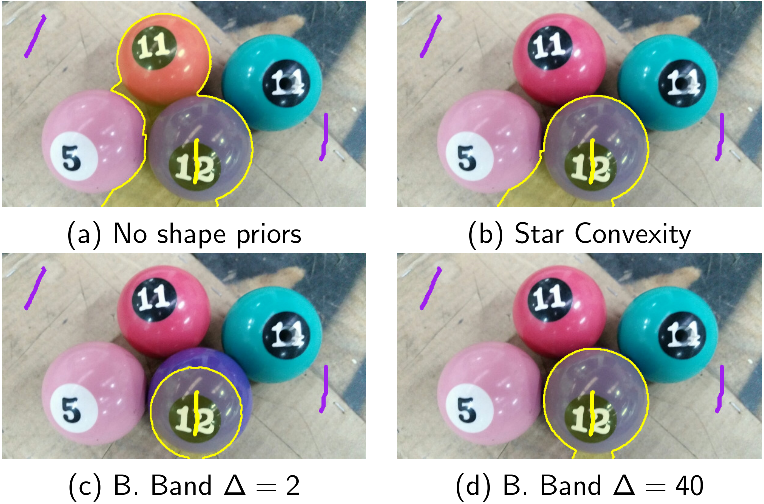


Figure: Pool ball OIFT segmentation with a circle template in a 600×338 image.

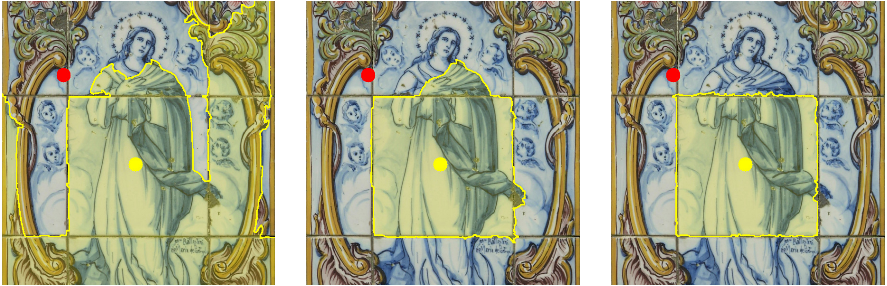
Circle Template

(e) Hedgehog $\theta = 45^\circ$

(f) Local Band $\Delta = 2$

Figure: Pool ball OIFT segmentation with a circle template in a 600×338 image.

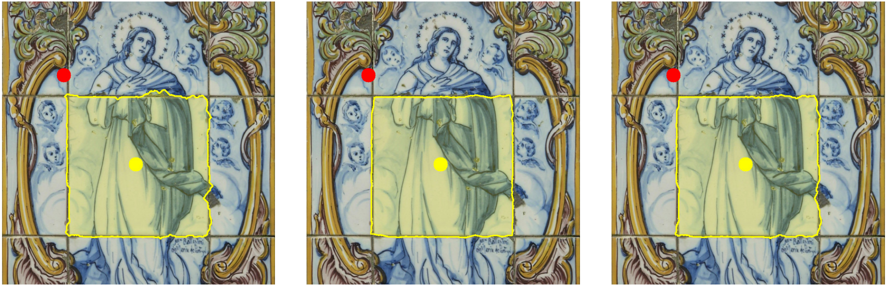
Square Template



(a) No shape priors (b) Star Convexity (c) B. Band $\Delta = 10$

Figure: Wall tile segmentation by OIFT with a square template in a 576×881 image.

Square Template



(d) Hedgehog $\theta = 45^\circ$ (e) Local Band $\Delta = 1$ (f) Local Band $\Delta = 2$

Figure: Wall tile segmentation by OIFT with a square template in a 576×881 image.

Seed Displacement

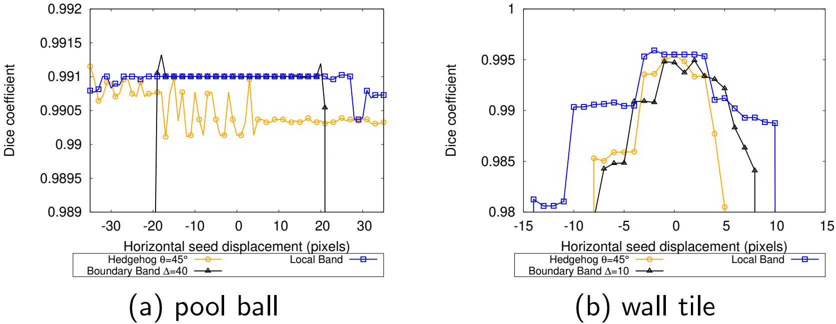
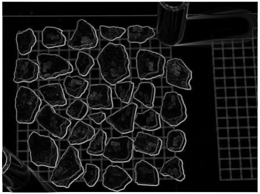
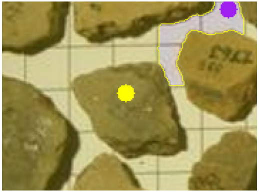


Figure: The accuracy curves for different horizontal displacements of the internal seeds.

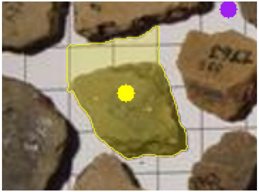
Archeological Fragments



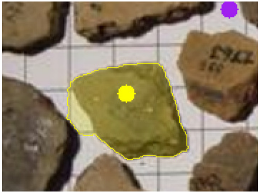
(a) Sobel gradient



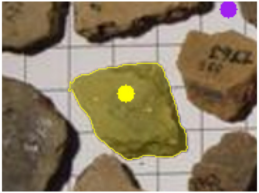
(b) No priors



(c) B. Band



(d) Hedgehog



(e) Local Band

Figure: Archaeological fragment segmentation.

Archeological Fragments - Results

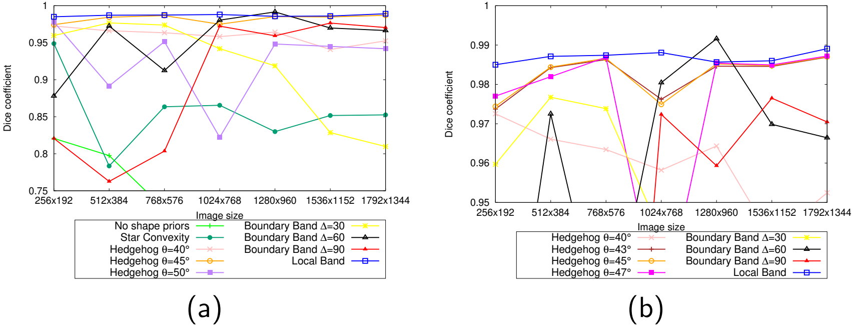
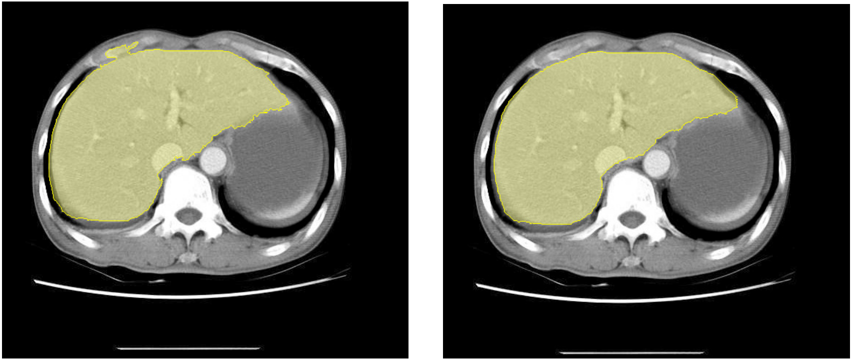


Figure: (a)The mean accuracy values to segment the archaeological fragments for different image resolutions.(b) Zoomed results (accuracy $\geq$ 95%).

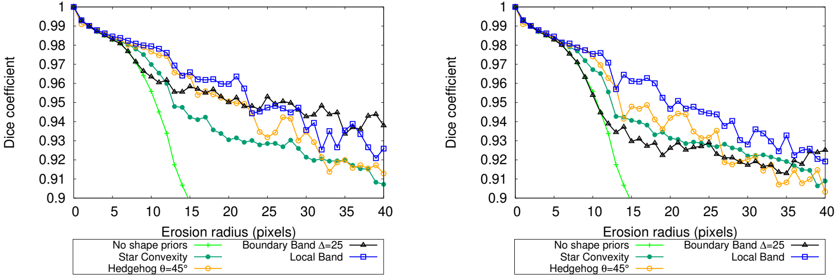
Liver Image Displacement



(a) B. Band (b) Local Band

Figure: The mean accuracy curves to segment the liver for seed sets obtained by erosion.

Liver Image Displacement - Results



(a) $\mathcal{S}_1$ centralized (b) $\mathcal{S}_1$ shifted to the left

Figure: The mean accuracy curves to segment the liver for seed sets obtained by erosion.

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Summary

We have proposed the Local Band shape constraint, for the GGC framework, which in its limit case (i.e., $R \to \infty$) is strongly related to Boundary Band constraint and is less sensitive to the seed/template positioning.

To the best of our knowledge, we are also the first to report OIFT with the Hedgehog shape prior.

THANKS!