

Cardano formula (1570). The solutions of the equation $x^3 + px + q = 0$ are of the form

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}.$$

Our example. Consider the equation $x^3 - 3x + 1 = 0$. We have $p = -3$ and $q = 1$, so

$$\sqrt{\frac{q^2}{4} + \frac{p^3}{27}} = \frac{\sqrt{3}}{2}i.$$

Thus the roots are of the form

$$x = \sqrt[3]{\frac{-1 + i\sqrt{3}}{2}} + \sqrt[3]{\frac{-1 - i\sqrt{3}}{2}}.$$

You need to be careful in interpreting the cubic roots of complex numbers. Each of the above cubic roots has three values, but having a value for one of them, we have only one choice for the other.